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GEOMETRICAL EXERCISES

FOR BEGINNERS

BY

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TO

THE REV. J. H. JELLETT, D.D.

PROVOST OF TRINITY COLLEGE, DUBLIN,

THIS WORK IS DEDICATED,

WITH UNFEIGNED PLEASURE,

BY THE AUTHOR,

AS A SLIGHT PROOF OF GRATITUDE FOR MUCH KINDNESS

RECEIVED, AND AS A TRIBUTE TO HIS GENIUS

AND SCHOLARSHIP.

PREFACE.

THE following Exercises are submitted to the student in the hope that they may be of use both in assisting him to understand those propositions with which he must subsequently become familiar, and in enabling him to solve problems for himself by means of the Examples which have been given throughout, and to which hints have been added which the Author trusts will be found judicious.

The only works on the subject with which the Author is acquainted treat so much of Modern and so little of Ancient Geometry, that he has reason to hope the present one will not be superfluous.

Many of the proofs which have been adopted are original, though those usually given have not been departed from except when the change was considered to be an advantage.

The pupil is advised first to try those propositions which have been worked out in full, before he reads their solutions, for even should he fail to obtain a solution his time will not have been wasted. In solving a problem he should in every case first *suppose* it solved. Prop. 35

may be looked on as an example of this. Those Examples (pages 130, 131) which depend chiefly on Euclid (I. 4; I. 8; I. 26) had better be solved before the Exercises are begun.

The Author begs to thank his friends Messrs Nicolls, B.A., Senior Moderator, Trinity College, Dublin, and L. J. Ryan, Head Master, Central Model Schools, Dublin, for their kindness in correcting the proof-sheets.

19, ST ANDREW STREET, DUBLIN.

ERRATA.

Page 16, sixth and seventh lines from the bottom, *for* AHG *read* AHC.

Page 54, last line, *for* O *read* O_1.

Page 59, line 13, *for* AE *read* BE, *for* AF *read* BF.

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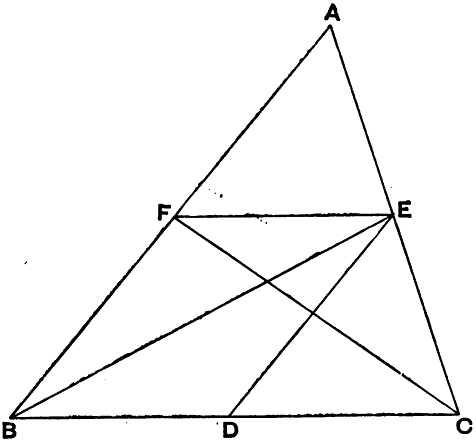
GEOMETRICAL EXERCISES

FOR BEGINNERS.

FUNDAMENTAL PROPOSITIONS.

PROP. 1. The straight line joining the middle points of the sides of a triangle is parallel to the third side and equal to half of it.

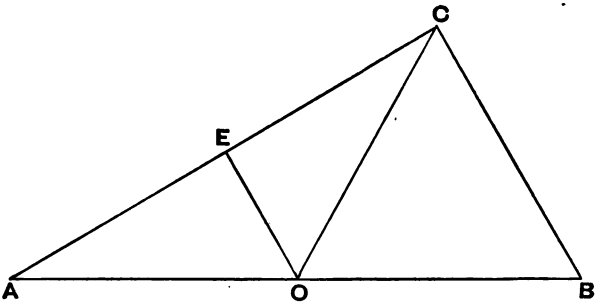


Let D, E, F, be the middle points of the sides BC, CA, AB, of any triangle ABC. Join FE, BE, CF, and ED.

As the triangles *AFE, BFE* stand upon equal bases *AF, BF,* and are between the same parallels, since they have a common vertex, *E,* they are equal (I. 38). For the same reason the triangles *AFE, CFE,* are equal; therefore the triangles *BFE, CFE* are equal. But they stand upon the same base, *FE,* and upon the same side of it; therefore they are between the same parallels (I. 39). Hence *FE* is parallel to the side *BC.* Similarly the line *ED* is parallel to the side *AB*; therefore *BFED* is a parallelogram, and therefore (I. 34) *FE, BD* are equal: that is, *FE* is equal to half the base.

Note. The converse of this very important theorem may be stated thus:—If, through the middle point of one side of a triangle, a parallel be drawn to the base, it bisects the other side.

PROP. 2. Half the hypotenuse of a right-angled triangle is equal to the straight line joining its middle point to the vertex.



Let *ABC* be a right-angled triangle, of which *AB* is the hypotenuse. Bisect *AB* in *O.* Join *CO*: the straight lines *AO, BO, CO* are equal.

Bisect *AC* in *E*; and join *OE.*

Since *O*, *E*, are the middle points of *AB*, *AC*, respectively, *OE* is parallel to *BC* (I.); therefore (I. 29) the angles *AEO*, *ACB* are equal; but *ACB* is a right angle, therefore *AEO* is a right angle. Hence in the triangles *AEO*, *CEO*, the sides *AE*, *EO* are equal to the sides *CE*, *EO*, each to each, and the contained angles *AEO*, *CEO* are also equal, each being right; therefore (I. 4) the bases *AO*, *CO* are equal. But *AO*, *BO* are equal, since *O* is the middle point of *AB*; therefore the three *AO*, *BO*, *CO* are equal.

PROP. 3. If from any two points, and the middle point of the straight line joining them, any three parallel lines be drawn meeting a third indefinite straight line: (1) the sum of the two external parallels is double the internal one, when the two points are on the same side of the indefinite line: (2) the difference of the external parallels is double the internal one, when the two points are on different sides of the indefinite line.

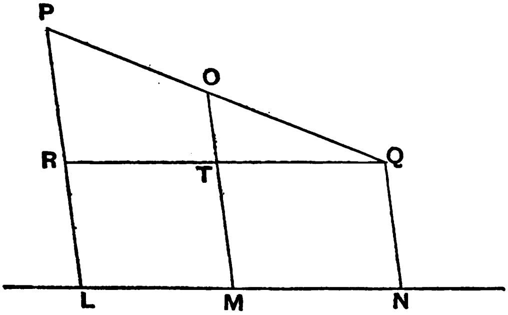


Fig. 1

Let *P*, *Q* be the two points; *LMN* the indefinite straight line; *O* the middle point of *PQ*. Draw *PL*, *QN*, *OM*, any three parallel lines meeting the indefinite straight line in *L*, *M*, *N*, respectively: the sum of *PL*, *QN* (fig. 1)

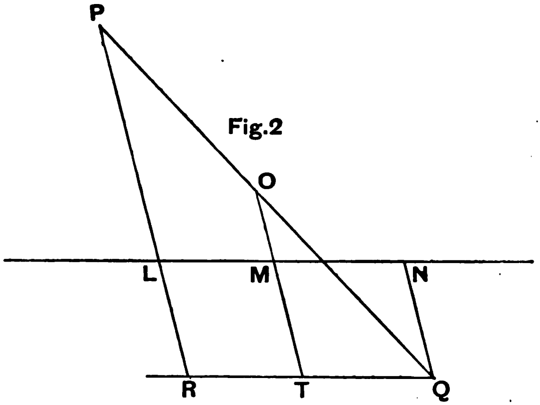


Fig.2

is double *OM*: and the difference of *PL*, *QN* (fig. 2) is double *OM*.

Through *Q* draw *QR*, parallel to *LMN*, and meeting *PL*, *OM*, produced if necessary, in *R*, *T* respectively.

Since *O* is the middle point of *PQ*, and *OT* is parallel to the base *PR* of the triangle *PQR*; therefore

$$(1, \textit{Note})\ 2OT = PR.$$

And since the opposite sides of a parallelogram are equal, $2TM = QN + RL$; ∴ by addition (fig. 1)

$$2(OT + TM) = PR + RL + QN;$$

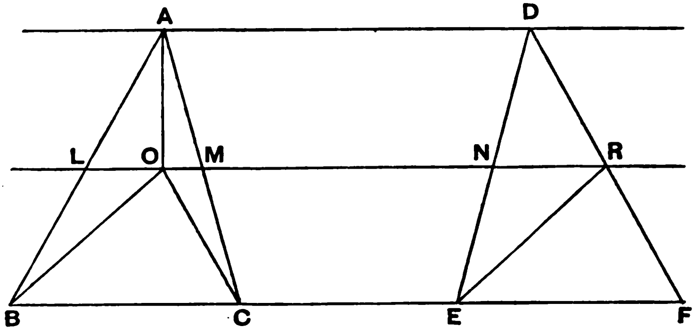
that is, $2OM = PL + QN$: which proves (1).

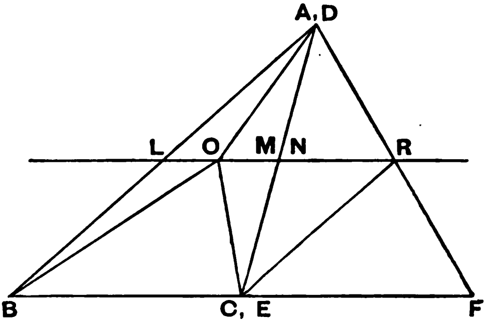
Again by subtraction (fig. 2)

$$2(OT - TM) = PR - RL - QN;$$

that is, $2OM = PL - QN$; which proves (2).

Prop. 4. If any straight line be drawn parallel to the common base line, and cutting the sides, of two triangles standing upon equal (or the same) bases and between the same parallels, the sides of the triangles intercept equal segments on this line.





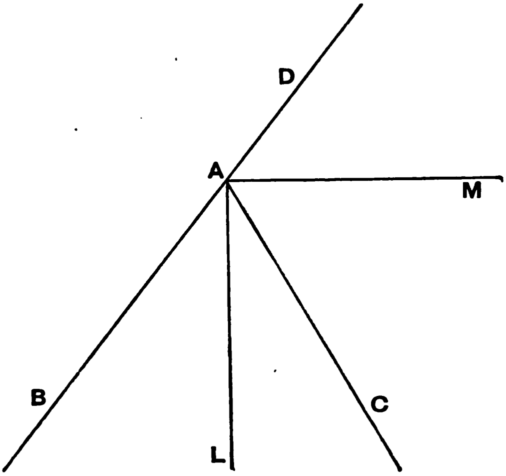
Let *ABC*, *DEF* be two triangles upon equal bases *BC*, *EF*, and between the same parallels *AD*, *BF*. Let *LMNR* be any straight line parallel to *BF*, and cutting the sides respectively, in *L*, *M*, *N*, *R*: then the intercepts *LM*, *NR* are equal.

For, if not, suppose one of them *LM* to be the greater. Cut off *LO* equal to *NR*. Join *AO*, *BO*, *CO*, and *ER*.

The triangles AOL, DNR, being upon equal bases and between the same parallels, are (I. 38) equal. Similarly the triangles BOL, NER, and BOC, ERF, are equal; therefore the triangle DEF is equal to the figure $ABCOA$. But the triangles DEF, ABC are equal (I. 38); therefore the triangle ABC is equal to the figure $ABCO$, which is absurd. Hence LM is not greater than NR. Similarly NR is not greater than LM; therefore they are equal.

Note. The above proof applied to fig. 2, evidently proves the important theorem, that "the straight line joining the vertex of a triangle to the middle point of the base bisects every parallel to the base."

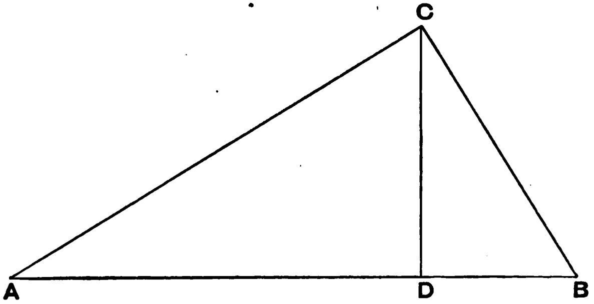
PROP. 5. The internal and external bisectors of any angle are at right angles to each other.



Let BAC be any angle. Produce BA to any point D. Bisect BAC internally by the line AL, and externally (that is, the angle *CAD*) by *AM*: then *LAM* is a right angle.

The sum of the angles *BAC*, *CAD* is (I. 13) equal to two right angles; therefore half their sum is one right angle; but *CAL* is half of *BAC*, and *CAM* is half of *CAD*; therefore the angle *LAM* is half the sum of the angles *BAC*, *CAD*, that is, the angle *LAM* is a right angle.

PROP. 6. The angle between either side of a right-angled triangle and the perpendicular from the vertex to the hypotenuse is equal to the angle between the hypotenuse and the other side.

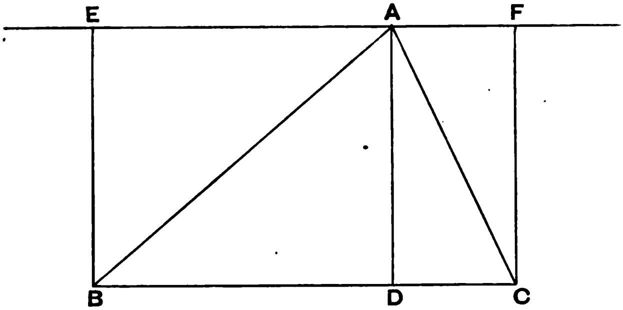


Let *ABC* be a right-angled triangle, of which *AB* is the hypotenuse. Draw *CD* perpendicular to *AB*: the angles *BCD*, *BAC* are equal: as are, also, the angles *ACD*, *ABC*.

The angle *CDB* is equal to the sum of the angles *BAC*, *ACD* (I. 32). But the angle *CDB* is right by construction; therefore the sum of the angles *BAC*, *ACD* is equal to a right angle.

Again, the sum of the angles *ACD*, *BCD* is a right angle by hypothesis; therefore the sum of the angles *BAC*, *ACD*, is equal to the sum of the angles *ACD*, *BCD*. Hence, taking away the common angle *ACD*, the angles *BCD*, *BAC* remain equal. Similarly the angles *ACD*, *ABC* are equal.

Prop. 7. The area of a triangle is equal to half the rectangle contained by its base and altitude.



Let *ABC* be any triangle; and from the vertex, *A*, draw *AD* perpendicular to the base *BC*: the area of the triangle *ABC* is equal to half the rectangle *BC*.*AD*.

Through *A* draw *EAF* parallel to the base *BC*. Through *B*, *C* draw *BE*, *CF* parallel to *AD*, and meeting *EAF* in *E*, *F*, respectively.

Since *AD* is perpendicular to *BC*, and *BE*, *CF* are parallel to *AD*, clearly *BEFC* is a rectangle.

But (I. 41) the rectangle *BEFC* is double the triangle *ABC*; therefore *BC*.*CF* is double the triangle *ABC*. But *CF*, *AD* are equal, being the opposite sides of a rectangle; hence *BC*.*AD* is double the area of *ABC*, or, the area of the triangle *ABC* is equal to ½*BC*.*AD*.

EXAMPLES.

1. Prove that the triangle AFE in Prop. 1 is equal to one fourth the area of the triangle ABC.

2. If the middle points of the sides of a quadrilateral be joined in order, the figure thus formed is a parallelogram; and its area is half that of the quadrilateral.

[Consider Prop. 1.]

3. The straight lines joining the middle points of the opposite sides of a quadrilateral bisect each other.

[This follows immediately from the last, if it can be shewn that the diagonals of a parallelogram bisect each other; which latter I. 26 will at once prove.]

4. An indefinite number of triangles stand upon the same base and between the same parallels; find the locus of the middle points of their sides.

[It will be found from Prop. 1 that the several middle points lie on a straight line midway between the parallels, which straight line is the locus of the points.]

5. If two triangles stand upon the same base, and not between the same parallels, the figure formed by joining the middle points of their sides is a parallelogram.

6. If from any two diametrically opposite points on the circumference of a circle perpendiculars be drawn to a straight line outside the circle, the sum of these perpendiculars is constant.

[Consider Prop. 3.]

7. A straight line of given length slides between two other straight lines at right angles; find the locus of its middle point.

[Since (2) half the hypotenuse of a right-angled triangle is equal to the distance of its middle point from the vertex; therefore, no matter what its position be, provided its length is always the same, the distance of its middle point from the fixed vertex is constant; in other words, its middle point moves on the circumference of a circle whose centre is the vertex, and whose radius is half the given length.]

8. If, from the vertex of a right-angled triangle, a perpendicular be drawn to the hypotenuse, the angle included between this line and that joining the vertex to the middle point of the hypotenuse is equal to the difference of the base angles.

[Consider Props. 2, 6.]

9. Given of a right-angled triangle the hypotenuse and the difference of the base angles; construct it.

[By means of the preceding example and Prop. 2, the solution is manifest.]

10. If the base of a triangle be divided into any number of equal parts, the straight lines joining the vertex to the several points of section divide every straight line drawn parallel to the base into the same number of equal parts.

[This follows immediately from *Note*, Prop. 4.]

11. If from the ends of the base of any triangle perpendiculars be drawn to the external bisector of the vertical angle, and their feet joined transversely to the ends of the base; prove that the joining lines intersect on the internal bisector of the vertical angle.

[Produce the perpendiculars to meet the sides produced. The solution will then become immediately apparent from Prop. 5, and Prop. 4, *Note.*]

12. State, and prove, the analogous theorem to the preceding.

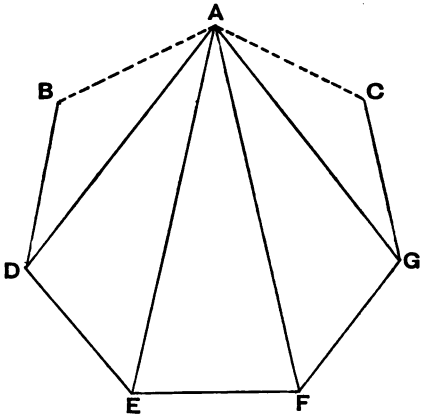
13. If perpendiculars be drawn from any point within an equilateral triangle to the sides; prove that their sum is constant, and is equal to the altitude of the triangle.

[Consider Prop. 7 for the solution of this and the next two examples.]

14. If perpendiculars be drawn from any point within an equilateral polygon to the sides; prove that their sum is constant.

15. Prove that the rectangle contained by the two sides of a right-angled triangle is equal to the rectangle contained by the hypotenuse and the perpendicular.

PROP. 8. The sum of the internal angles of a polygon of n sides is equal to $2(n-2)$ right angles.



Let *ABDEFGC* be a polygon of any number of sides, n. Join any one vertex, A, to the several vertices D, E, etc. Then the polygon is made up, clearly, of as many triangles as the figure has sides, *less by two*: that is, the number of triangles is $n-2$. But (I. 32) every triangle has the sum of its angles equal to two right angles; therefore the sum of the angles of the $n-2$ triangles, or the sum of the internal angles of the polygon, is equal to $2(n-2)$ right angles.

EXAMPLES.

1. The sum of the internal angles of a polygon is equal to thirty-six right angles; find the number of its sides.

Let n be the number of sides, then

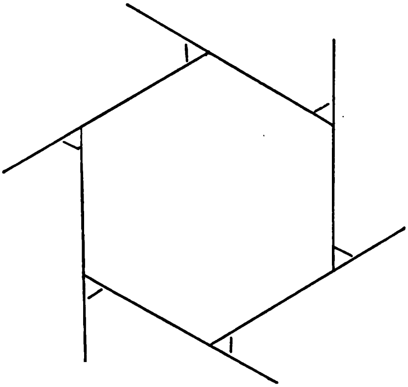
$$2(n-2)=36,$$
$$\text{or } 2n=40,$$
$$\text{or } n=20.$$

2. The sum of the internal angles of a polygon is equal to eighteen right angles; prove that it must have eleven sides.

3. Prove that there are sixteen right angles in a decagon.

4. Prove that the sum of the internal angles of any polygon can never be an odd number of right angles.

PROP. 9. If the sides of a polygon of any number of sides be produced successively, the sum of the external angles is equal to four right angles.

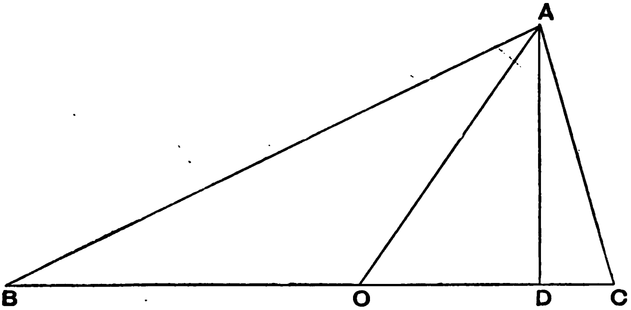


Since the sum of any one internal and the adjacent external angle of the polygon is equal to two right angles

(I. 13); therefore the sum of the external together with the internal angles is equal to $2n$ right angles, n being the number of its sides.

Hence, the sum of the external angles is equal to the difference between $2n$ and $2n-4$ right angles: which is, by common subtraction, 4 right angles.

Prop. 10. If from the vertex of a right-angled triangle a perpendicular be drawn to the hypotenuse, it divides it into segments, the rectangle under which is equal to the square of the perpendicular.



Let ABC be a triangle, right-angled at A.

Draw AD perpendicular to BC:

the rectangle under BD, DC is equal to the square on AD.

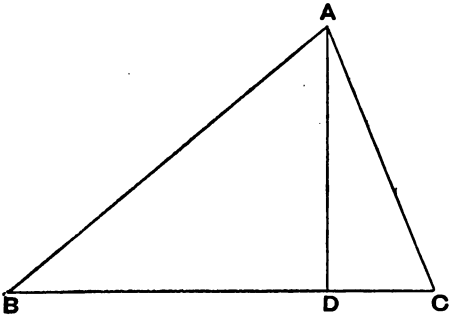
Bisect BC in O, and join AO.

Clearly (II. 5) $BD.DC+OD^2=OC^2=OA^2$, since (2) $OA=OC$; to each side add AD^2,

and $$BD.DC+OD^2+AD^2=OA^2+AD^2;$$

but $OA^2=OD^2+AD^2$ (I. 47); therefore, taking away these equals, we have $BD.DC=AD^2$.

PROP. 11. The difference of the squares of the sides of a triangle is equal to the difference of the squares of the segments of the base made by the perpendicular from the vertex on the base.



Let ABC be any triangle. Draw AD perpendicular to BC:

the difference of the squares of AB, AC is equal to the difference of the squares of BD, DC.

Since the angle ADB is right,

$AB^2 = AD^2 + BD^2$ (I. 47).

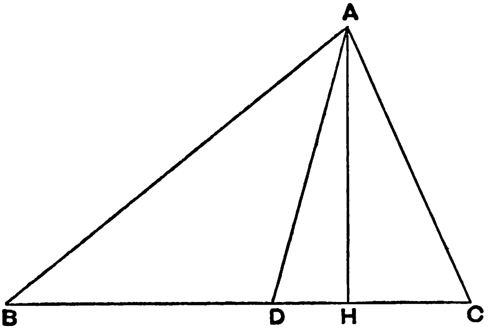
Similarly $AC^2 = AD^2 + CD^2$;

$\therefore$ by subtraction $AB^2 - AC^2 = BD^2 - CD^2$.

PROP. 12. The sum of the squares of the sides of a triangle is equal to double the square of the bisector of the base together with double the square of half the base.

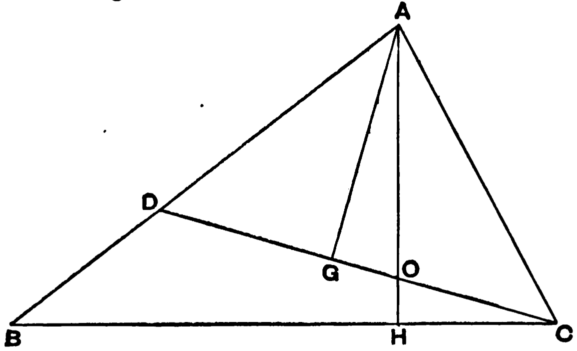
Let ABC be any triangle, of which D is the middle point of the base BC, and AH the perpendicular from the vertex on the base. Join AD.

Since the angle ADB is obtuse, we have (II. 12) $AB^2 = AD^2 + BD^2 + 2BD \,.\, DH$: and since the angle ADC



is acute, we have (II. 13) $AC^2 = AD^2 + DC^2 - 2CD \,.\, DH$; therefore, by addition, $AB^2 + AC^2 = 2AD^2 + 2BD^2$; for, BD being equal to CD, their squares add, and the rectangles cancel.

Prop. 13. If from the greater of two sides of a triangle a portion be cut off equal to the less, and the point of section joined to the vertex of the opposite base angle, this line makes with the base an angle equal to half the difference of the base angles.



Let *ABC* be a triangle, of which *AD* is equal to *AC*, and *CD* joined: the angle *BCD* is equal to half the difference of the base angles.

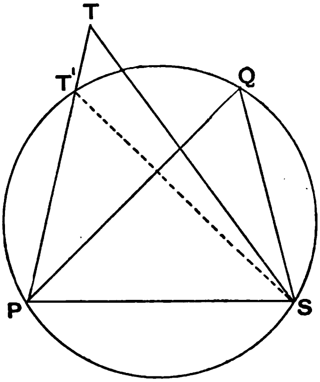
The angles *ADC*, *ACD* are equal (I. 5); and the angle *ADC* is (I. 32) equal to the sum of the angles *ABC*, *BCD*; therefore the angle *ACD* is equal to the sum of the angles *ABC*, *BCD*. To each of these equals add the angle *BCD*, and we have the sum of the angles *ACD*, *BCD* (or *ACB*) equal to the angle *ABC* together with twice *BCD*. That is to say, twice the angle *BCD* is equal to the difference of the angles *ACB*, *ABC*.

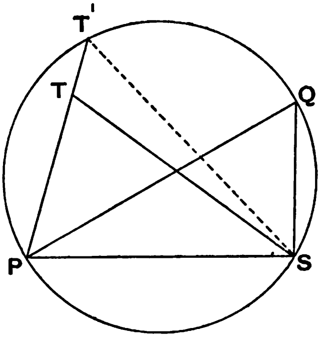
PROP. 14. The angle, between the bisector of the vertical angle of a triangle and the perpendicular from the vertex to the base, is equal to half the difference of the base angles.

The construction being the same as in the last, let *AG*, the bisector of the angle *A*, meet *CD* in *G*; and let *AH*, the perpendicular from the vertex to the base, cut *CD* in *O*.

Since the lines *AD*, *AC* are equal, the angles *ADC*, *ACD* are equal; and the angles *DAG*, *CAG* are equal by construction; therefore, since the side *AG* is common to the two triangles *DAG*, *CAG*, the angles *AGD*, *AGC* are equal (I. 26); and therefore (Euclid def.) each of these angles is a right angle. But the angle *AHG* is also right; therefore the angles *AHG*, *AGC* are equal: and the angles *AOG*, *HOC* are equal (I. 15); therefore (I. 32) the angles *GAO*, *HCO* are equal. But *HCO* is, by the preceding, half the difference of the base angles; therefore the angle *GAO* is half the difference of the base angles.

Prop. 15. If two triangles, having their vertical angles equal, stand upon the same base and upon the same side of it, they have the same circumscribing circle.





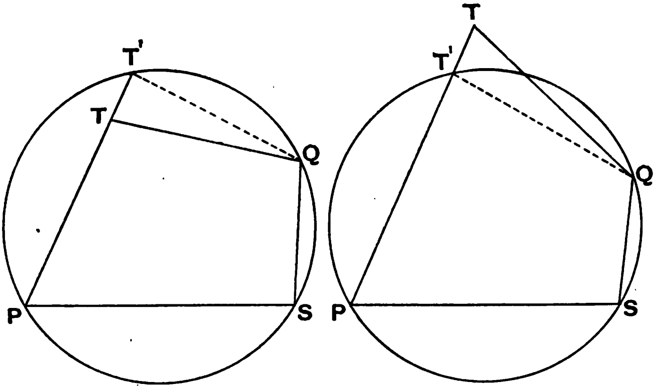
Let *PQS*, *PTS* be two triangles, having their vertical angles *PTS*, *PQS* equal, and standing upon the same base *PS*: the same circle will pass through the points *P*, *T*, *Q*, *S*.

For, if not, let the circumscribing circle of *PQS* not pass through *T*, but cut *PT*, produced if necessary, in *T'*. Join *ST'*.

Because the angles *PT'S*, *PQS* are in the same segment they are equal (III. 21). But (hyp.) the angles *PTS*, *PQS* are equal; therefore the angles *PTS*, *PT'S* are equal, which contradicts (I. 16). Hence the points *P*, *T*, *Q*, *S* must lie on the same circle.

Prop. 16. If the opposite angles of a quadrilateral are together equal to two right angles, it is circumscribable by a circle.

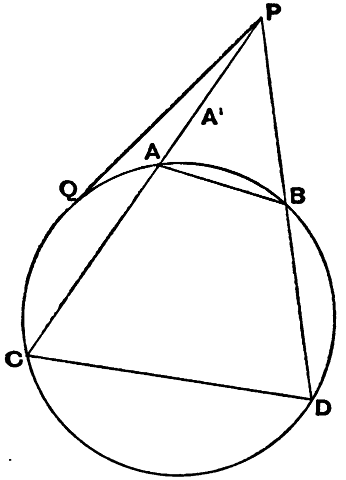
Let *PTQS* be a quadrilateral, having the sum of the opposite angles *PTQ*, *PSQ* equal to two right angles: the same circle will pass through the points *P*, *T*, *Q*, *S*.



For, if not, let the circumscribing circle of the triangle *PSQ* not pass through *T*, but cut *PT*, produced if necessary, in *T'*. Join *QT'*.

Since the quadrilateral *PT'QS* is inscribed in a circle, the sum of the angles *PT'Q*, *PSQ* is equal to two right angles (III. 22). But (hyp.) the sum of the angles *PTQ*, *PSQ* is equal to two right angles; therefore the angles *PTQ*, *PT'Q* are equal, which contradicts (I. 16). Hence the circle passing through *P*, *S*, *Q*, also passes through *T*.

PROP. 17. *ABCD* is a quadrilateral inscribed in a circle, if the opposite sides *AC*, *BD* are produced to meet in *P*, the rectangle under *PA*, *PC* shall be equal to the rectangle under *PB*, *PD*; and the angles *PBA*, *PCD* shall be equal: conversely, if the rectangle under *PA*, *PC* is equal to the rectangle under PB, PD; or the angles PBA, PCD are equal, the quadrilateral is circumscribable by a circle.



Draw PQ a tangent to the circle.

Then $PA \,.\, PC = PQ^2$. [III. 36.]

Similarly $PB \,.\, PD = PQ^2$.

$\therefore PA \,.\, PC = PB \,.\, PD$.

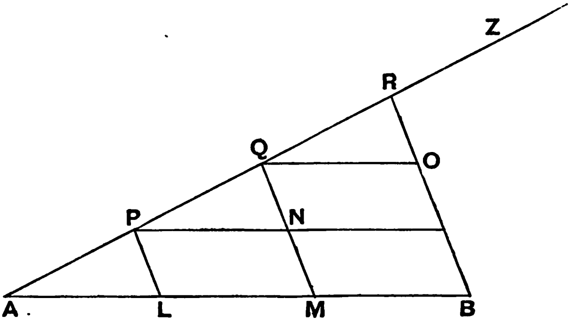
Again, (III. 22) the sum of the angles ABD, ACD is equal to two right angles; but (I. 13) the angles PBA, ABD together make two right angles; therefore the angles PBA, ABD are together equal to the angles PCD, ABD; hence, taking away the common angle ABD, the angles PBA, PCD are equal.

Conversely, if $PA \,.\, PC = PB \,.\, PD$; or the angles PBA, PCD are equal; $ABCD$ is circumscribable by a circle.

Suppose, in the first case, that the circle passing through *B*, *C*, *D* does not pass through *A*; let it cut *PC* in *A′*. Then, from the first part, $PA'.PC=PB.PD$; but (hyp.) $PA.PC=PB.PD$, $\therefore$ $PA'.PC=PA.PC$, and $\therefore$ $PA=PA'$, which is absurd. Hence the circle passing through *BCD* also passes through *A*.

In the second case, when the angles *PBA*, *ACD* are equal: to each of these equals add the angle *ABD*, when the sum of the angles *PBA*, *ABD* is equal to the sum of the angles *ABD*, *ACD*; but the former sum is (I. 13) equal to two right angles; therefore the latter sum is equal to two right angles; and therefore (16) the quadrilateral *ABCD* is circumscribable by a circle.

Prop. 18. To divide a given finite straight line into any number of equal parts.



Let *AB* be the given finite straight line: it is required to divide it into any number of equal parts.

From *A* draw any straight line *AZ*, and produce this line indefinitely. Upon *AZ* take any point *P*. Cut off (I. 3) *PQ*, *QR*, etc., each equal to *AP*, taking as many equal parts, *AP*, *PQ*, *QR*, on *AZ*, as we want to divide *AB* into.

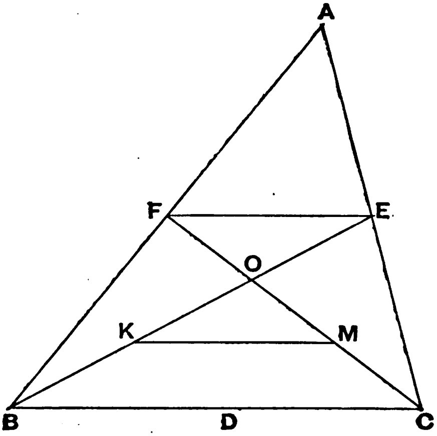
Join *RB*. Draw *PL*, *QM* parallel to *RB*: the parts *AL*, *LM*, *MB* are equal. Draw *QO*, *PN* parallel to *AB*, and meeting *RB*, *QM* in *O* and *N* respectively.

In the triangles *APL*, *PQN*, the sides *AP*, *PQ* are equal; and the angles *PQN*, *QPN* equal respectively to the angles *APL*, *LAP*, (I. 29); therefore (I. 26) the sides *AL*, *PN* are equal. Similarly the sides *PN*, *QO* are equal; and so on; therefore the lines *AL*, *PN*, *QO*, etc. are equal. But the lines *PN*, *QO*, are (I. 34) equal to *LM*, *MB*, respectively; therefore *AL*, *LM*, *MB* are equal.

In this special case the line *AB* is trisected.

Note. The proof of this problem shews that "if one side of a triangle be divided into any number of equal parts, and if parallels to the base be drawn from the several points of section, these parallels will divide the other side into the same number of equal segments."

Prop. 19. The straight lines drawn from the ends of



the base of a triangle to the middle points of the opposite sides trisect each other at their point of intersection.

Let *ABC* be any triangle, and *E*, *F* the middle points of *AC*, *AB*, respectively. Join *CF*, *BE*, intersecting in *O*. Bisect *OB*, *OC*, in *K*, *M*, respectively. Join *KM*, *FE*.

Since *F*, *E* are the middle points of *AB*, *AC* respectively, *FE* is parallel to *BC*, and equal to half of it (1). And considering the triangle *BOC*, *KM* is parallel to *BC*, and equal to half of it (1). Therefore the lines *KM*, *FE* are both equal and parallel. Hence, in the triangles *OFE*, *OKM*, the angles *FEO*, *EFO* are equal to the angles *OKM*, *KMO*, respectively, and the bases *FE*, *KM* are equal; therefore (I. 26) the lines *OE*. *OK* are equal, and the lines *FO*, *OM* are equal. But *OK*, *KB* are equal by construction; therefore the three *EO*, *OK*, *KB* are equal; and similarly the three *FO*, *OM*, *MC* are equal. Hence the lines *CF*, *BE* trisect each other, the greater segments being towards the angles.

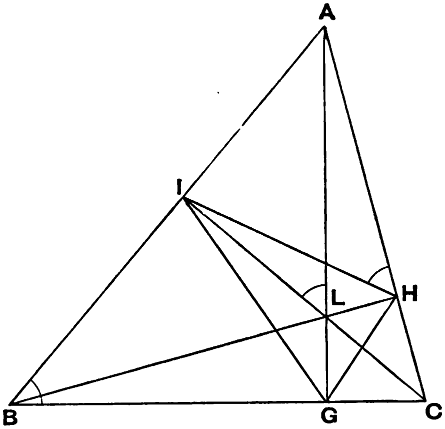
Prop. 20. The straight lines joining the vertices of a triangle to the middle points of the opposite sides (generally called—the bisectors of the sides,) meet in a point.

Let (last fig.) *ABC* be any triangle, and *D*, *E*, *F*, the middle points of the sides *BC*, *CA*, *AB*, respectively: *BE*, *CF*, *AD* meet in a point.

Join *BE*. Let (18) *O* be one point of trisection of the line *BE*, *BO* being the greater segment.

By the preceding the line drawn from any vertex, *C*, to the middle point of the opposite side *AB* must pass through *O*. Similarly the line drawn from *A* to the point *D* must pass through *O*. Hence the three lines meet in a point.

PROP. 21. The perpendiculars from the vertices of a triangle to the opposite sides meet in a point: and if a triangle be formed by joining their feet, its sides will be equally inclined to those of the original triangle.



Let *ABC* be any triangle; and let the perpendiculars *BH*, *CI*, intersect in *L*. Join *AL*, and let it, produced, meet *BC* in *G*: *AG* is perpendicular to *BC*.

Join *IH*. Since the angles *AIC*, *AHB* are each right, the quadrilateral *AILH* is circumscribable by a circle (16); therefore (III. 21) the angles *ALI*, *AHI* are equal. Again, since the angles *BHC*, *BIC* are equal, each being right, the quadrilateral *BIHC* is circumscribable by a circle; therefore (17) the angles *AHI*, *ABC* are equal; but the angles *ALI*, *AHI* have been proved equal; therefore the angles *ALI*, *ABC* are equal; and, therefore, the quadrilateral *BILG* is circumscribable by a circle. But the angle *BIL* is a right angle, therefore (III. 22) the angle *AGB* is a right angle, in other words, *AG* is perpendicular to *BC*.

To prove the second part. Join *IG*, *HG*.

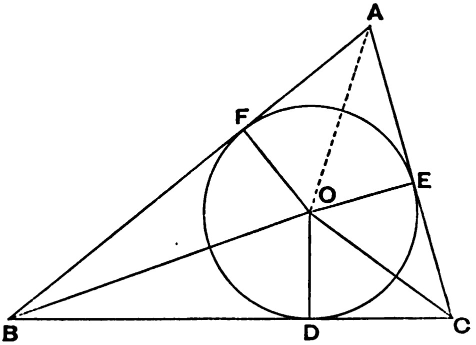
The angles *BLI*, *CLH* are equal (I. 15); but these angles are equal, respectively, to the angles *BGI*, *CGH*, since the quadrilaterals *BGLI*, *CGLH* are circumscribable by circles; therefore the latter angles *BGI*, *CGH* are equal, which proves the second part.

Examples.

1. Prove that the point, *L*, in the preceding proposition, is the centre of the inscribed circle of the triangle *GHI*.

2. Prove that the sides *BC*, *CA*, *AB* of the triangle *ABC* are the external bisectors of the angles of the triangle *GHI*.

Prop. 22. The internal bisectors of the angles of a triangle meet in a point—the centre of the inscribed circle.



Let *ABC* be any triangle. Bisect the angles *B*, *C*, by the lines *BO*, *CO*, meeting in *O*. Join *AO* : *AO* bisects the angle *A*.

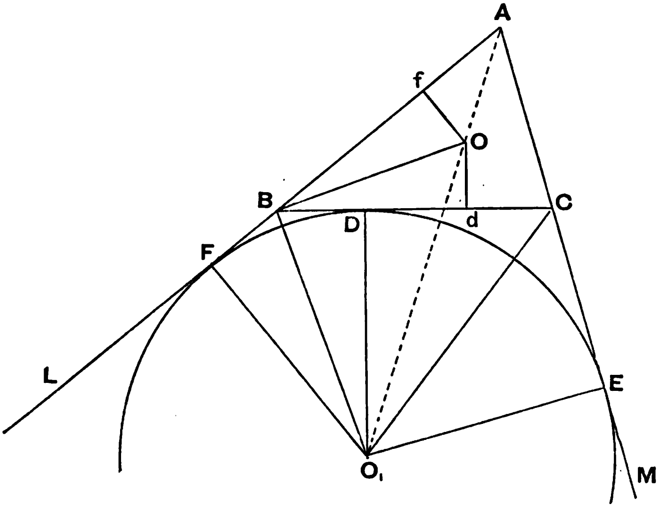
From *O* draw *OD*, *OE*, *OF*, perpendicular to *BC*, *CA*, *AB*, respectively.

In the triangles BOF, BOD, the angles BFO, FBO are equal, respectively, to the angles BDO, DBO; and the side BO common; therefore (I. 26) the lines OD, OF are equal.

Similarly the lines OD, OE are equal; therefore the three OD, OE, OF are equal: whence O is the centre of the inscribed circle.

Again, (I. 47) $AO^2 = AF^2 + FO^2 = AE^2 + OE^2$; therefore $AF^2 + FO^2 = AE^2 + OE^2$, and taking from these equals the equal squares of FO, OE, the squares AF, AE remain equal. Hence, in the triangles AOF, AOE, the sides AF, FO are equal to the sides AE, EO, respectively, and the side AO common; therefore (I. 8) the angles FAO, EAO are equal, that is, AO bisects the angle A.

PROP. 23. The external bisectors of any two angles of a triangle and the internal bisector of the third angle meet in a point—called the centre of an escribed circle.



Let ABC be a triangle; and let the sides AB, AC be produced indefinitely to L, M, respectively. Bisect the angles CBL, BCM by the lines BO_1, CO_1, respectively, meeting in O_1. Join AO_1: AO_1 is the internal bisector of the third angle A.

Draw O_1D, O_1E, O_1F perpendicular to BC, and the production of AC, AB, respectively.

In the triangles O_1BD, O_1BF, the angles O_1BD, O_1DB, are equal, respectively, to the angles O_1BF, O_1FB, and the side O_1B common; therefore (I. 26) the sides O_1D, O_1F are equal.

Similarly the sides O_1D, O_1E, are equal, therefore the three O_1D, O_1E, O_1F, are equal. (Hence the circle, with O_1 as centre, and with radius either of these three lines, touches the side BC externally, and the other two sides AB, AC, produced. Wherefore the point O_1 is called the centre of an escribed circle.) Again, (I. 47)

$$AO_1^2 = AF^2 + O_1F^2 = AE^2 + O_1E^2.$$

But O_1E^2, O_1F^2 are equal; therefore AF^2, AE^2 are equal; and therefore the lines AF, AE are equal.

Hence, in the triangles AO_1F, AO_1E, the sides AF, FO_1 are equal to the sides AE, EO_1, respectively, and the side AO_1 common; therefore (I. 8) the angles FAO_1, EAO_1 are equal, that is, O_1A is the internal bisector of the angle A.

EXAMPLES.

1. If a, b, c, denote the lengths of the sides BC, CA, AB, respectively, of a triangle ABC; and if $2s = a + b + c$: prove that $AF = AE = s - a$; $BF = BD = s - b$; $CD = CE = s - c$; D, E, F, being the points of contact of the inscribed circle.

2. If D, E, F, are the points of contact of the escribed circle opposite the angle A: prove that $AF = AE = s$; $BF = BD = s - c$; $CD = CE = s - b$.

3. Prove that the middle point of $O\ O_1$ is equally distant from the vertices, B, C, of any triangle ABC; O, O_1 being the centres of the inscribed, and escribed circle opposite the angle A.

[Consider Props. 2 and 5.]

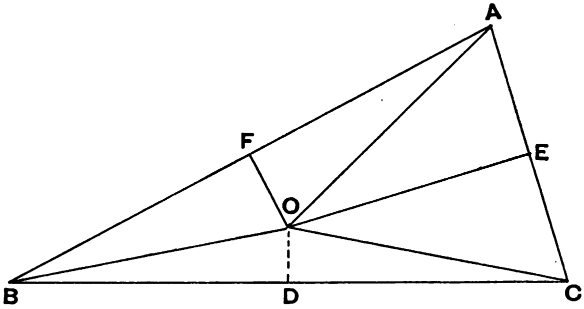
4. The area of a triangle is equal to sr, r being the radius of the inscribed circle.

[For the solution of this and the next example, consider Prop. 7.]

5. The area of a triangle is equal to $(s - a)\,r_1$, r_1 being the radius of the escribed circle opposite the angle A.

6. If d, D are the points of contact of the circles (O), (O_1) on the side a; prove that the distance dD is equal to the difference of the other two sides c, b.

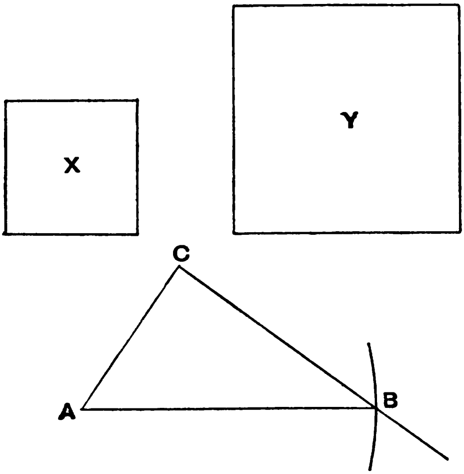
PROP. 24. The perpendiculars to the sides of a triangle at their middle points meet in a point—the centre of the circumscribing circle.



Let D, E, F be the middle points of the sides BC, CA, AB, respectively, of a triangle ABC. Let the perpendiculars *FO*, *EO*, at *F*, *E*, meet in *O*. Join *OD*: *OD* is perpendicular to *BC*.

Join *OA*, *OB*, *OC*. In the triangles, *AFO*, *BFO*, the sides *AF*, *FO* are equal, respectively, to the sides *BF*, *FO*, and the contained angles *AFO*, *BFO* are also equal; therefore (I. 4) the bases *AO*, *BO* are equal. Similarly the bases *AO*, *OC* are equal; therefore the three lines *OA*, *OB*, *OC* are equal—whence *O* is the centre of the circumscribing circle. In the triangles *OBD*, *ODC*, the sides *BD*, *DO* are equal to the sides *CD*, *DO*, respectively, and the bases *BO*, *OC* are also equal; therefore (I. 8) the angles *ODB*, *ODC* are equal; and, therefore, *OD* is at right angles to *BC*.

PROP. 25. To construct a square equal to
(1) the sum of two given squares:
(2) the difference of two given squares:
(3) double a given square.



Let X, Y, be the two given squares.

(1) To find a square equal the sum of these two given squares.

Take AC equal to a side of the square X, and BC equal to a side of the square Y. Place these lines at right angles at the point C; and join AB: the square on AB is the required one.

For, (I. 47) the square on AB is equal to the sum of the squares on AC, CB, that is, the sum of the squares X, Y.

(2) To find a square equal to the difference of the squares X, Y.

Lay down the line AC equal to a side of the square X, and at C erect CB perpendicular to AC, and produce it indefinitely. With A as centre, and a radius equal to a side of the square Y, describe a circle cutting CB in B. Join AB. The square on CB is the required square.

For, the square on AB being (I. 47) equal to the sum of the squares on AC, CB; therefore the square on CB is equal to the difference of the squares on AB, AC, that is, the difference of the squares Y, X.

(3) To find a square equal to double a given square.

We have only to take the squares X, Y equal, when the proof of (1) clearly applies.

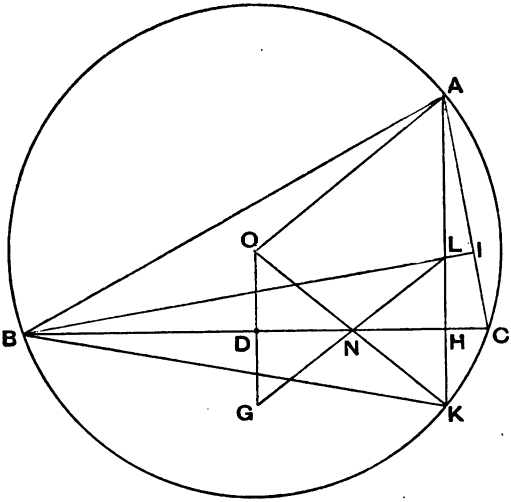
EXAMPLE.

Construct a square that shall be equal to the sum of three given squares.

[First construct a square equal to the sum of any two of them, by means of (1). Then construct a square equal to the sum of the one just found and the third given square: clearly this last is the required one.

By this method, a square can be found equal to the sum of any number of given squares.]

PROP. 26. If the perpendiculars of a triangle be produced to meet the circumference of the circumscribing circle, the intercepts, between their point of intersection and the circumference, are bisected by the sides of the triangle.



Let *ABC* be a triangle, of which *AH*, *BI* are the perpendiculars from *A*, *B*, respectively, to *BC*, *CA*. Let *AH*, produced, meet the circumference in *K*: then *LK* is bisected in *H*, *L* being the intersection of the perpendiculars.

Join *BK*.

Since the angles *AIB*, *AHB* are each right, a circle will circumscribe *AIHB* (15); therefore (III. 21) the angles *HBL*, *IAL* are equal. Again, the angles *CAK* (or *IAL*), *CBK* are equal, being in the same segment (III. 21); therefore the angles *CBK*, *HBL* are equal. But the angles *LHB*, *KHB*, are equal, being right (const.): hence, in the triangles *BHL*, *BHK*, the side *BH* is common, and the angles *HBL*, *BHL* are equal, respectively, to the two, *KBH*, *KHB*; therefore (I. 26) the sides *LH*, *HK* are equal, that is, *LK* is bisected in *H*. Similarly it can be shewn that the other intercepts are bisected.

PROP. 27. The distance of any vertex of a triangle from the intersection of the perpendiculars is double the perpendicular from the centre of the circumscribing circle upon the opposite side.

The construction being the same as in the preceding, draw from *O*, the centre of the circumscribing circle, *OD* perpendicular to *BC*. Join *OK*, cutting *BC* in *N*. Join *LN*, meeting *OD*, produced, in *G*. Join *OA*.

Since, by the last, *LH*, *HK* are equal, we have in the triangles *NLH*, *NHK*, the sides *NH*, *HL* equal to the sides *NH*, *HK*, respectively, and the included angles *NHL*, *NHK* equal, each being right; therefore (I. 4) the angles *LNH*, *HNK* are equal, as are, also, the angles *NLH*, *NKH*. But the angles *NKH*, *OAK* are equal, since *OA*, *OK* are equal; therefore the angles *NLH*, *OAK* are equal: hence (I. 28) *OA*, *LG* are parallel. But *AH*, *OG*, are parallel, since each is perpendicular to *BC*; therefore *AOGL* is a parallelogram, wherefore (I. 34) *AL*, *OG* are equal.

Again, it has been proved that the angles *LNH*, *KNH* are equal; therefore their equals *DNG*, *DNO* are equal.

Hence in the triangles *DNG*, *DNO*, the side *DN* is common, and the angles *DNG*, *NDG*, equal, respectively, to the angles *DNO*, *NDO*; therefore (I. 26) the lines *OD*, *DG* are equal: that is, *OG* is double of *OD*. But *OG*, *AL*, have been proved equal; therefore *AL* is double of *OD*.

EXAMPLE.

A straight line of given length slides between the legs of a fixed angle; find the locus of the intersection of the perpendiculars of the triangle thus formed.

[The radius of the circumscribing circle is clearly constant, (III. 24): and since the base, also, is given, the perpendicular from the centre to it is, therefore, given. Hence, from the above proposition, the locus is a circle, with the vertex of the fixed angle as centre, and a radius of given length.]

PROP. 28. Squares are described upon the sides of any triangle, and perpendiculars are drawn to the base from the two most remote edges; then, the sum or difference of these perpendiculars is equal to the base, according as both base angles are each acute, or one base angle obtuse.

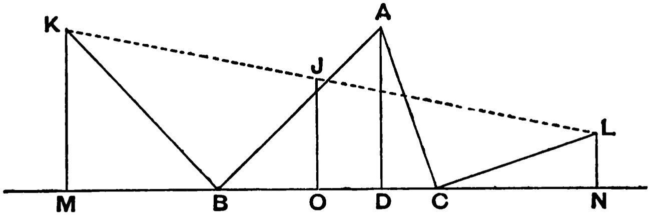


Fig. I

Let *ABC* be any triangle. At *B*, *C* erect *BK*, *CL* perpendiculars to *AB*, *AC*, and equal to these lines respectively. Then *K, L* are the edges of the two squares most remote. Draw *KM, LN* perpendiculars to *BC* produced. Draw, also, *AD* perpendicular to *BC*, or *BC* produced.

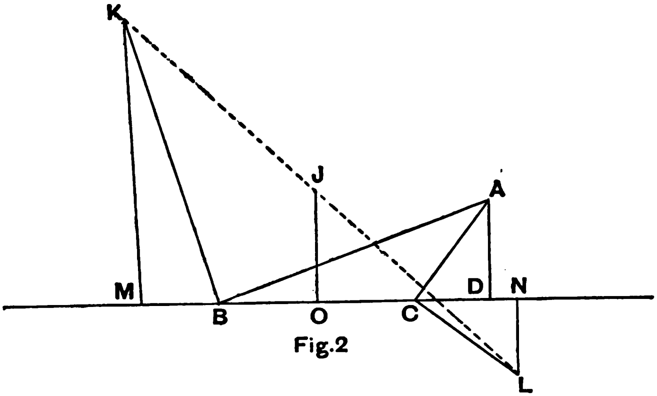


Fig. 2

Since the angle *ABK* is right, being the angle of a square, the angles *ABD, KBM* together make a right angle. But the angles *BAD, ABD* also make a right angle, since the angle *ADB* is right; therefore the angles *KBM, BAD* are equal; and the angles *KMB, ADB* are each right: hence, in the triangles *KMB, ADB*, the sides *AB, BK* are equal by construction, and the angles *KBM, KMB* are equal to the angles *BAD, ADB*, each to each; therefore (I. 26) the lines *KM, BD* are equal. In a similar manner, the lines *CD, LN* can be proved equal. Hence (fig. 1), where each base angle is acute, clearly, the sum of *KM, LN* is equal to the sum of *BD, CD*, that is, *BC*.

In (fig. 2) the case where one base angle is obtuse, clearly the difference between *KM, LN* is equal to the difference between *BD, CD*, that is, *BC*.

Prop. 29. Given the base of a triangle in magnitude and position, to prove that the straight line joining the two most remote edges of the squares described upon the sides always passes through a fixed point.

Let (figs. to preceding) *ABC* be any triangle upon the fixed base *BC*: the straight line joining *K*, *L*, passes through a fixed point.

Bisect *BC* in *O*. Erect *OJ* perpendicular to *BC*, and meeting *KL* in *J*. By the last, the triangles *KBM*, *ABD* are equal in every respect. Hence the lines *BM*, *AD* are equal. For the same reason, the lines *CN*, *AD* are equal: therefore the lines *BM*, *CN* are equal. Hence since *O* is the middle point of *BC*, it is also the middle point of *MN*. But the three lines *MK*, *OJ*, *NL* are parallel; therefore (3) the sum (or difference) of *MK*, *LN* is equal to double *OJ*. But, by the last proposition, the sum (or difference) of *MK*, *LN* is equal to *BC*; therefore *BC* is double *OJ*; or the lines *BO*, *OJ* are equal. Hence, *OJ* is always constant, while the base is constant; but it is at right angles also to the base at its middle point, therefore the point *J* is fixed. Hence the several straight lines *KL* pass through this point.

Prop. 30. If a straight line be drawn from the

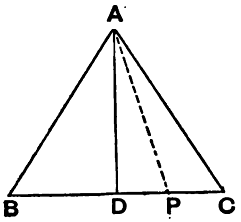


Fig. 1

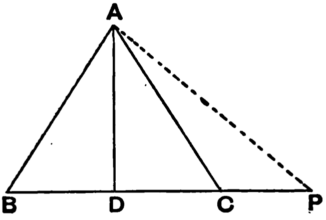


Fig. 2

vertex of an isosceles triangle to any point in the base, the rectangle under the segments of the base, thus made, is equal to the difference of the squares of a side of the triangle and the joining line.

Let ABC be any isosceles triangle. Take any point P in the base, or base produced: the rectangle

$$BP \,.\, PC = AB^2 - AP^2$$

when the point is on the base; and

$$BP \,.\, PC = AP^2 - AB^2$$

when the point is on the base produced.

Draw AD at right angles to BC. Then, since the angles ABC, ACB are equal, and the angles at D right, and the side AD common to the triangles ABD, ADC, the sides BD, DC are equal. (I. 26.)

Hence (fig. 1) $BP \,.\, PC = BD^2 - DP^2$ (II. 5);

but (11) $BD^2 - DP^2 = AB^2 - AP^2$,

therefore $BP \,.\, PC = AB^2 - AP^2$.

And (fig. 2) $BP \,.\, PC = DP^2 - CD^2$ (II. 6)

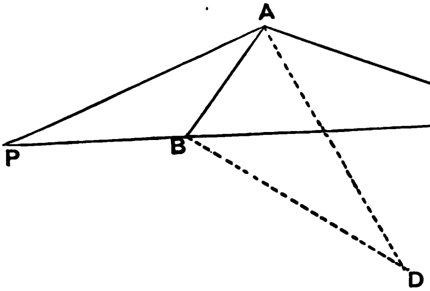
$= DP^2 - BD^2$;

but (11) $DP^2 - BD^2 = AP^2 - AB^2$,

therefore $BP \,.\, PC = AP^2 - AB^2$.

PROP. 31. If from the vertex, A, of a triangle ABC, a straight line AP be drawn making the angle BAP equal to the angle ACB, and cutting the base CB, produced, in P: then the line AP is a tangent to the circumscribing circle of the triangle ACB: and $CP \,.\, PB = PA^2$.

Let the angles BAP, ACB be equal: then AP is a tangent to the circle circumscribing the triangle ACB: and $CP.PB = PA^2$.



At A erect AD perpendicular to AP, and at B erect BD perpendicular to AB; let these perpendiculars meet in D.

Since the angle ABD is right, the sum of the angles BAD, ADB is equal to a right angle (I. 32); but the angle DAP is a right angle; therefore the sum of the angles BAD, ADB is equal to the sum of the angles BAD, BAP; take away the common angle BAD, and the angles ADB, BAP are equal. But the angles BAP, ACB are equal by hyp.; therefore the angles ADB, ACB, are equal; and therefore (15) the same circle will pass through the points A, B, D, C. But the angle ABD is right; therefore AD is a diameter of the circle $ABDC$; and it is at right angles to AP; therefore (III. 16) AP is a tangent to the circumscribing circle of ABC: and therefore (III. 36) $CP.PB = PA^2$.

EXAMPLE.

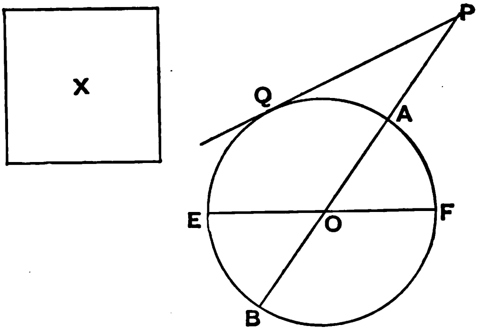
From the vertex, A, of a triangle, draw a straight line AP cutting the base BC, produced, in P, so that $BP\,.\,PC$ shall be equal to the square on AP. When would the problem be impossible?

PROP. 32. The rectangle contained by the hypotenuse of a right-angled triangle and either segment of the hypotenuse made by the perpendicular is equal to the square of the adjacent side.

The construction being the same as in Prop. 6: it is required to prove that $AB.BD=BC^2$.

Since the angles BCD, BAC are equal (6), BC is a tangent to the circle about the triangle ACD (31). Therefore (III. 36) $AB.BD=BC^2$.

PROP. 33. Given the rectangle under two lines, and the difference between them: to find the lines.



It is required to find two lines, so that the rectangle under them shall be equal to a given square X, and the difference between them shall be equal to a given straight line EF.

Upon the given straight line EF, as diameter, describe a circle. At any point, Q, on this circle, draw a tangent, and cut off from it a portion, QP, equal to side of the given square X. Join P to O, the centre of the circle, and let PO cut the circumference in A, B; PA, PB are the required lines.

For (III. 36) $PA \cdot PB = PQ^2 =$ the given square X; and $PB - PA = AB = EF$ the given difference.

EXAMPLES.

1. If from any point, O, within a triangle ABC, perpendiculars OD, OE, OF be drawn to the sides BC, CA, AB respectively; prove that $BD^2 + CE^2 + AF^2 = CD^2 + AE^2 + BF^2$.

[Apply Prop. 11 to the several triangles OBC, OCA, OAB. It will be found by addition that the squares OB, OC, OA cancel.]

2. Prove that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of its diagonals.

[See Prop. 12. It is necessary, also, for the solution of this theorem and the next, to know that "the square on a line is equal to four times the square on its half." This the student can easily prove for himself. It is clearly a particular case of (II. 4.).]

3. The sum of the squares of the sides of a quadrilateral is equal to the sum of the squares of its diagonals, together with four squares of the line joining the middle points of the diagonals.

4. Shew that Ex. 2 is a particular case of Ex. 3.

5. Prove that (see fig. to Prop. 21)

$$AL \cdot AG = AH \cdot AC = AB \cdot AI.$$

And hence prove that the perpendiculars from the vertices of a triangle to the opposite sides meet in a point.

PROP. 34. Given, of a right-angled triangle, either side, and the remote segment of the hypotenuse made by the perpendicular: to construct it.

Suppose (fig. Prop. 6) ABC the required triangle, and that AC, BD are, respectively, the given side, and remote segment of the hypotenuse made by the perpendioular CD.

Since (32) $BA . AD = AC^2$, we know the rectangle under the lines BA, AD. But we also know their difference, viz., BD; hence (33) we can find the lines BA, AD.

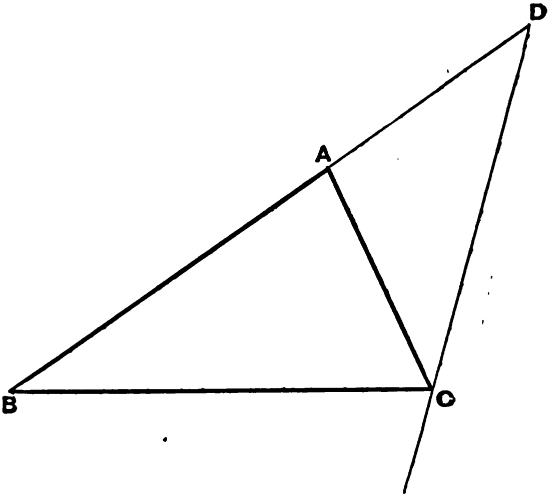
The following, then, is the construction:—Lay down the lines BA, AD (AD being taken as a portion of AB), the rectangle under which is equal to the square on the given side, and their difference being equal to the given difference. At D erect DC perpendicular to AB, and produce it indefinitely. Inflect over to this perpendicular the line AC, equal to the given side, and meeting DC in C. Join CB: ACB is the required triangle.

For, $BA . AD$ being equal to AC^2, AC is (III. 37) a tangent to the circumscribing circle of the triangle ABC; but CB is the diameter of this circle, since CDB is a right angle; therefore (III. 16) ACB is a right angle. The two other conditions of the problem are clearly satisfied by the construction.

PROP. 35. Given the base, vertical angle, and sum of sides of a triangle: to construct it.

Suppose that ABC is a triangle that satisfies the three conditions, viz. that its base BC is equal to a given line, that its vertical angle A is equal to a given angle, and that the sum of its sides is equal to a given line.

Produce *BA*, and take *AD* equal to *AC*. Join *CD*.



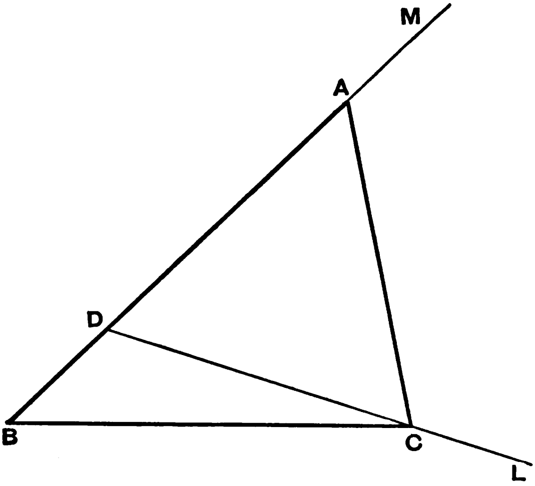
Since (I. 5) the angles *ACD*, *ADC* are equal, the angle *A* is (I. 32) double the angle *ADC*. Hence the angle *ADC* is half the given vertical angle. And *BD* is equal to the given sum of sides, since the lines *AD*, *AC* are equal. Hence the following construction :—

Lay down *BD* equal to the given sum of the sides; and at either end, *D*, make the angle *BDC* equal to half the given vertical angle.

Inflect over *BC* to the line *DC*, and equal to the given base. At *C* make the angle *DCA* equal to the angle *BDC*. Let the line *CA* cut *BD* in *A*: *ABC* is the required triangle.

For, since the angles *ADC*, *ACD* are equal, the angle *BAC* is double the angle *ADC*. But the angle *ADC* was made equal to half the vertical angle; therefore the angle BAC is equal to the given vertical angle. Again, the sides AC, AD are equal (I. 6); therefore the sum of BA, AC is equal to the given sum BD. And BC is clearly the given base.

PROP. 36. Given the base, vertical angle, and the difference of the sides of a triangle: to construct it.



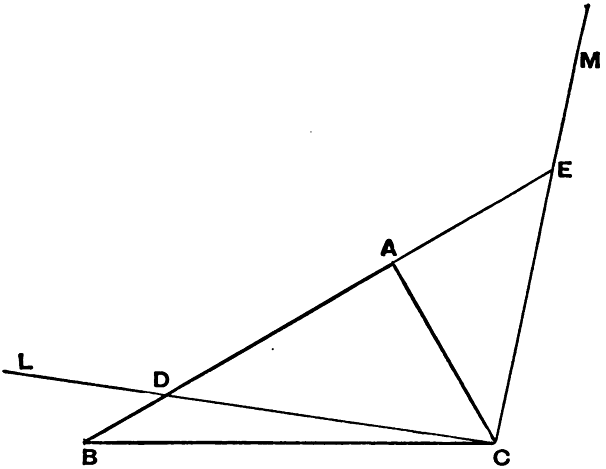
Suppose that the triangle ABC satisfies the three conditions of the problem. Then, cutting off AD equal to AC, BD is equal to the given difference of the sides; and since the angles ADC, ACD are equal, either of them, ADC, is (I. 32) half the supplement of the given vertical angle.

Hence the following construction:—Lay down BD equal to the given difference of the sides. Produce BD

indefinitely to *M*. Make the angle *MDL* equal to half the supplement of the given vertical angle. Inflect over *BC* to *DL*, and equal to the given base. At *C* make the angle *DCA* equal to the angle *ADC*: the triangle *ABC*, thus formed, is the required triangle.

For, the base *BC* is given by construction. And since the angles *ADC*, *ACD* are equal, the sides *AD*, *AC* are equal (I. 6); and, therefore, the difference between *AB*, *AC* is equal to *BD*, the given difference. And since the angles *ADC*, *ACD* were made half the supplement of the given vertical angle, and are half the supplement of the angle *A* from the geometry of the figure; therefore the angle *A* is the given vertical angle.

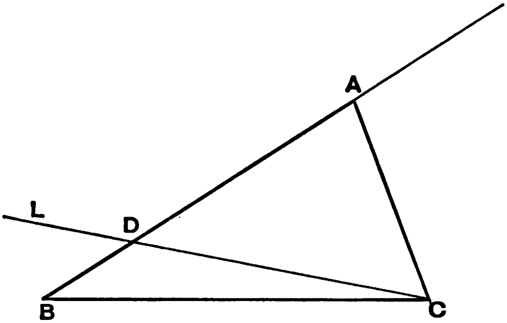
PROP. 37. Given the base, difference of the base angles, and sum of the sides of a triangle: to construct it.



The following is the construction (see Prop. 13). Lay down BC equal to the given base. Make the angle BCL equal to half the given difference of the base angles. At C erect CM at right angles to CL, and produce it indefinitely. Inflect over BE to CM, and equal to the given sum of the sides. Let BE cut CL in D. Bisect DE in A. Join AC: ABC is the required triangle.

For, its base is given by construction; and since DE is bisected in A, and DCE is a right angle, the three, DA, AC, AE are equal (2). Hence the sum of BA, AC is equal to the given sum BE. And since AD, AC, are equal, the angle BCL is half the difference of the base angle (13). But it was made half the given difference of the base angles; therefore the difference between ABC, ACB is given.

PROP. 38. Given the base, difference of the base angles, and difference of the sides of a triangle: to construct it.



The following is evidently the construction. (See Prop. 13.)

Lay down BC equal to the given base. Make the angle BCL equal to half the given difference of the base angles. Inflect BD over to CL, and equal to the given difference of the sides. Produce *BD* indefinitely. At the point *C*, with the straight line *CD*, make the angle *DCA* equal to the angle *CDA*, *CA* meeting *BD* produced in *A*:

ABC is the required triangle.

For, the base is given. And since the angles *CDA*, *DCA* are equal, the angle *BCD* is half the difference of the base angles (13); therefore the difference between the base angles is given. And clearly *BD* is equal to the given difference of the sides.

EXAMPLES.

1. Given of a right-angled triangle the perpendicular, and either of the base angles: construct it.

[Consider Prop. 6.]

2. Given the segments of the hypotenuse made by the perpendicular: construct the right-angled triangle.

3. Given of a right-angled triangle the hypotenuse and the sum, or difference, of the sides: construct it.

4. Given the three middle points of the sides of any triangle: construct it.

[Consider Prop. 1.]

5. Given, in position, the three feet of the perpendiculars of a triangle: construct it.

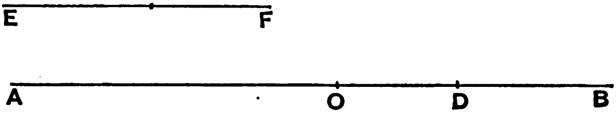
[Consider Prop. 21, and the example following.]

6. Given the three bisectors of the sides of a triangle: construct it.

7. Given the angles of a triangle, and the perimeter: construct it.

[Lay down the straight line, equal to the perimeter, as base line; upon it construct a triangle having its angles equal to the given angles (I. 23). From the centre of the inscribed circle of this triangle draw parallels to the sides: the small triangle thus formed will be the required one.]

PROP. 39. Given the sum and difference of two straight lines: to find them.

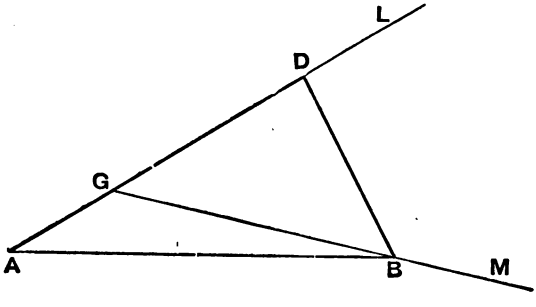


Let *AB, EF* be, respectively, the sum and difference of the two straight lines, which we are required to find.

Bisect *AB* in *O*, and from *OB* cut off (I. 3) the portion *OD* equal to half of the given difference *EF*: *AD, DB* are the required lines. For, their sum is clearly equal to *AB*; and since *AO, OB* are equal, the difference between *AD, DB* is equal to the difference between the sum of *OB, OD* and *DB*; or the difference between the sum of double *OD, DB* and *DB*; that is, double *OD*, or *EF*.

Note. From the above proof we see that *if a straight line be divided unequally, the difference of the segments is equal to double the distance of its middle point from the point of section.*

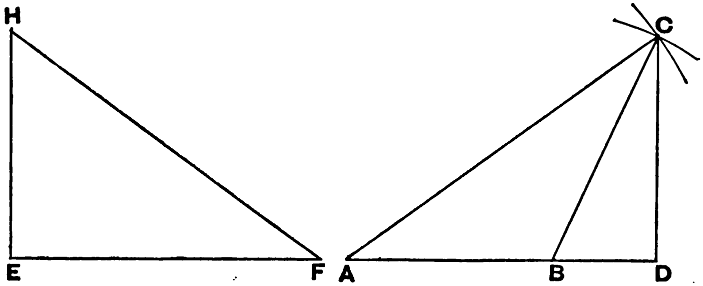
PROP. 40. Given the difference of two straight lines, and the sum of their squares: to find the lines.



Lay down *AG* equal to the given difference of the straight lines. Produce *AG* indefinitely to *L*. At the point *G*, with the straight line *GL*, make the angle *LGM* equal to half a right angle. Inflect over *AB* to the straight line *GM*, and equal to a side of the given square that is equal to the sum of the squares of the lines we are seeking. At *B* make the angle *GBD* equal to half a right angle; and let *BD* meet *GL* in *D*: *AD, BD* are the required lines.

For, since the angles *DGB, GBD* are each half a right angle, the angle *D* must be a right angle (I. 32), and the lines *DG, DB* must be equal. Hence (I. 47) the sum of the squares of *AD, DB* is equal to the given square on *AB*, and the difference of *AD, DB* is equal to the given difference *AG*.

PROP. 41. Given the difference of two straight lines, and the difference of their squares: to find the lines.



Let *AB* be equal to the difference between the lines we are seeking; and let *EF* be a side of the square that is equal to the difference of their squares.

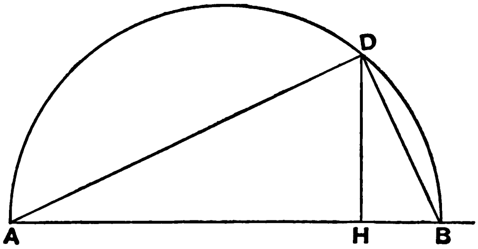
Construct any right-angled triangle *EFH* upon *EF*, having *HF* for hypotenuse. With *A* as centre, and a radius equal to *HF*, describe a circle. With *B* as centre, and a radius equal to *EH*, describe a circle. Let these circles intersect in *C*. From *C* draw *CD* perpendicular to *AB* produced: *AD*, *BD* are the required lines.

For, their difference is clearly equal to the given line *AB*, and the difference of their squares is (11) equal to the difference of the squares of *AC*, *CB*, that is, the difference of the squares of *HF*, *EH*, that is, the given square on *EF*.

PROP. 42. To divide a given straight line into two parts so that the difference of the squares of the segments shall be equal to a given square.

If we consider *AB*, in the preceding proposition, as the given straight line to be divided, the same proof as the last clearly applies: remarking, that the point *D* will be on the base, or the base produced, according as the given square on *EF* is less, or greater, than the square on *AB*.

PROP. 43. Given the difference of the squares of two lines, and the sum of their squares: to find the lines.

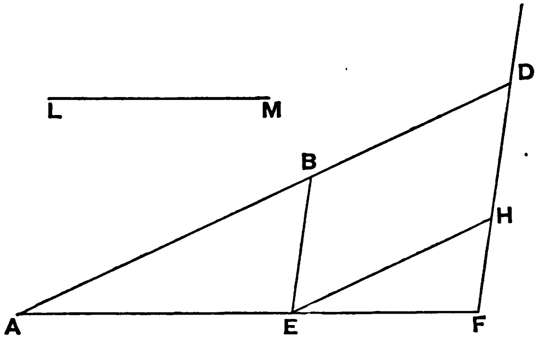


Let *AB* be a side of the square that is equal to the sum of the squares of the lines we are seeking. Upon

AB describe a semicircle. Divide (42) AB in H, so that $AH^2 - BH^2$ shall be equal to the given difference of the squares of the lines. At H erect HD perpendicular to AB, meeting the circumference of the semicircle in D. Join AD, BD : AD, BD are the required lines.

For, $AD^2 + BD^2 = AB^2$, since the angle, D, in a semicircle is right: and $AD^2 - BD^2 = AH^2 - BH^2$, since DH is a perpendicular. That is, the sum of their squares is equal to a given square, and the difference of their squares is equal to another given square.

PROP. 44. Given the difference of two straight lines, and their ratio: to find the lines.



Let LM be the given difference of the lines, and $AF : AE$ their given ratio. If EF is equal to LM, clearly, AF, AE are the required lines. If not, draw any line EH, equal to LM, and making any angle with EF. Join FH, and produce it indefinitely. Through A draw AD parallel to EH, meeting FH in D. Through E draw EB parallel to FD, and meeting AD in B: AD, AB are the required lines.

For, since the quadrilateral $EBDH$ is a parallelogram, BD, EH are equal; therefore BD is equal to the given line LM: and since BE, FD are parallel,

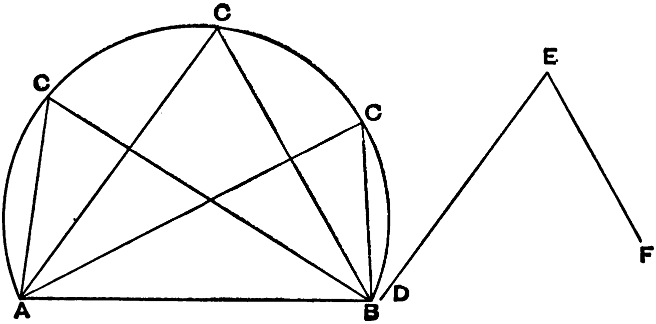
$$AD : AB :: AF : AE$$

(VI. 4), that is, in the given ratio.

ON LOCI, WITH THEIR APPLICATION TO THE CONSTRUCTION OF TRIANGLES.

Def. The locus of a point may be defined as being the straight line or circle to which the point is confined, and in any part of which it may lie, subject to certain conditions.

Prop. 45. Given the base, and vertical angle of a triangle: to find the locus of the vertex.



Let *AB* be the given base, and *DEF* the given angle: it is required to find the locus of the vertices of all triangles having this base, *AB*, and their vertical angles equal to *DEF*.

Upon *AB* describe (III. 33) the segment *ACB* capable of containing an angle equal to *DEF*: the arc *ACB* is the required locus.

For, taking any number of points, *C*, on this arc, and joining *CA*, *CB*, the several triangles *ACB* satisfy the two conditions, that they have a common base, and the same vertical angle *ACB* (III. 21), which vertical angle is equal to the given angle *DEF* by construction.

EXAMPLES.

1. The base *BC* of a triangle *ABC* is fixed, the vertical angle, *A*, is given; find the locus of the point, *D*, where *AD*, taken upon *BA* produced, is equal to *AC*.

2. Hence :—Given, of a triangle, the base, vertical angle, and the sum of the sides : construct it.

3. Find the locus of the point, *D*, where *AD* is taken upon *AB* not produced.

4. Hence :—Given, of a triangle, the base, vertical angle, and the difference of the sides : construct it.

5. A quadrilateral *ABCD* is inscribed in a circle; one side *BC* is fixed, while the opposite side *AD* is variable, but always of constant length; find the locus of the intersection (1) of *BD*, *AC*; (2) of *BA*, *CD*.

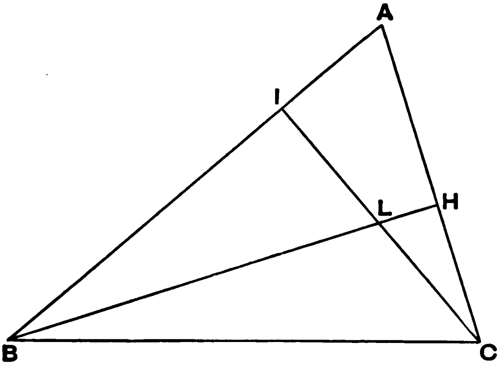
[If *BD*, *AC* intersect in *P*, it will be found without any difficulty that the angle *BPC* is a constant angle, whence the problem is reduced to Prop. 45.]

PROP. 46. Given the base, and vertical angle of a triangle: to find the locus of

(*a*) the intersection of the perpendiculars:
(*b*) the intersection of the bisectors of the angles:
(*c*) the intersection of the bisectors of the sides.

(*a*) Let *ABC* be a triangle, of which the base *BC* is fixed, and the angle *A* is given, but varying in position:

it is required to find the locus of the point *L*, where the perpendiculars *BH*, *CI*, intersect. The quadrilateral

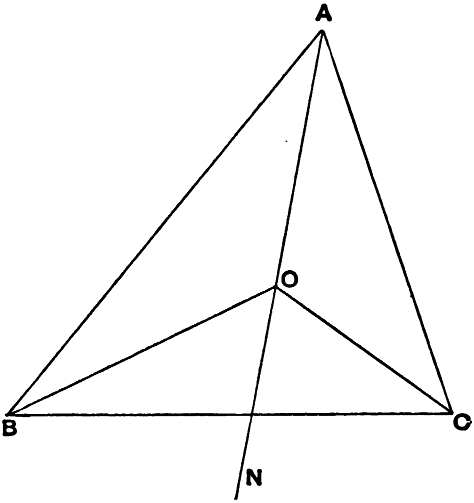


AILH is clearly circumscribable by a circle; therefore (III. 22) the sum of the angles *A*, *ILH* is equal to two right angles. But, since the angle *A* is given, the angle *ILH* must be given, that is, equal to the supplement of the angle *A*. But the angle *BLC* is equal to the angle *ILH*; therefore the angle *BLC* is equal to a given angle, the supplement of *A*. But the base *BC* is fixed; therefore, by the preceding, the locus of *L* is a segment of a circle upon *BC* as base, and containing an angle the supplement of the angle *A*.

(*b*) Let *BC*, as before, be the fixed base, and *BAC* the given vertical angle. Bisect the angles *B*, *C*, *A*, and let the bisectors meet in *O*: it is required to find the locus of this point.

Produce *AO* indefinitely to *N*.

The angle *BON* is (I. 32) equal to $\frac{A}{2}+\frac{B}{2}$; and the angle CON is equal to $\frac{A}{2}+\frac{C}{2}$; therefore the angle BOC

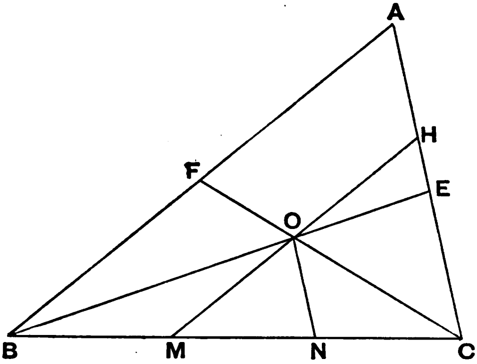


is equal to $A+\frac{B}{2}+\frac{C}{2}$. But, since (I. 32) $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}$ is equal to a right angle, the angle BOC is, therefore, equal to a right angle together with $\frac{A}{2}$. Hence the locus of O is a segment of a circle upon BC as base, and containing an angle equal to a right angle together with half the given vertical angle.

(*c*) Let, as before, BC be the given base, A the given vertical angle. Let BE, CF, the bisectors of the sides CA, AB, meet in O: it is required to find the locus of this point.

Draw OM, ON parallel to AB, AC respectively. Produce OM to meet AC in H.

Since (19) *OF* is the third part of *CF*, and *OM* is drawn parallel to *BF*; therefore (18, Note) *BM* is the third part of *BC*. Similarly *CN* is the third part of *BC*. Hence *MN* is a fixed segment.

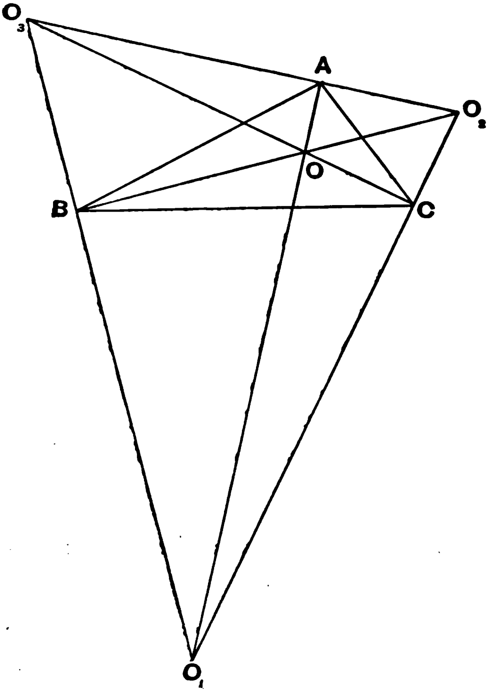


Again, the angles *MON*, *OHC* are equal (I. 29), and the angles *OHC*, *BAC* are also equal for the same reason; therefore the angle *MON* is equal to the given vertical angle *BAC*. But the base *MN* is fixed; therefore the locus of *O* is a segment of a circle upon *MN* as base, and containing an angle equal to the given vertical angle.

Prop. 47. Given the base and vertical angle of a triangle: to find the loci of the centres of the escribed circles.

Let *ABC* be a triangle, of which the base *BC* is fixed, and the vertical angle *A* is given.

Let *BAC* be one position of the triangle, having *BC* for fixed base and the angle *BAC* equal to the given vertical angle. Let the internal bisector of the angle *A*, and the external bisectors of the angles *B*, *C*, meet in *O* (see Prop. 23). Similarly find the points O_2, O_3: it is required, then, to find the loci of these three points.



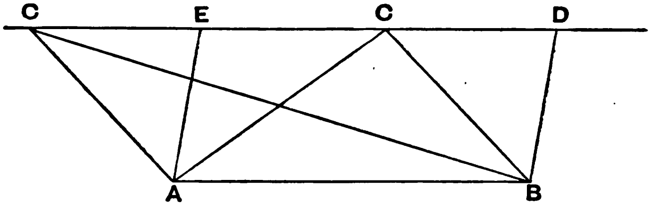
Since the external and internal bisectors of any angle are at right angles to each other (5), the vertices A, B, C, of the triangle ABC, are the feet of the perpendiculars of the triangle $O_1O_2O_3$. Again (see Ex. 1, page 24), the centre, O, of the inscribed circle, is the intersection of the perpendiculars, BO_2, CO_3, AO_1 of the triangle $O_1O_2O_3$. We have, then, the quadrilateral $AOCO_2$ circumscribable by a circle; therefore (III. 21) the angles OO_2C, OAC are equal, that is, the angle OO_2C is equal to half the given vertical angle.

Similarly the angle OO_3B is equal to half the given vertical angle. Hence the locus of the points O_2, O_3, is clearly the same arc of the segment of the circle described upon BC as base, and containing an angle equal to half the vertical angle.

To find the locus of the point O_1. The quadrilateral OBO_1C being circumscribable by a circle, the angles O_2OC, BO_1C are equal (17). But the angle O_2OC is equal to the angle O_2AC, that is, the complement of half the given vertical angle, therefore the angle BO_1C is given, and equal to the complement of half the given vertical angle.

Hence, the locus of the point O_1 is the arc of the segment of the circle upon BC as base, and containing an angle equal to the complement of half the given vertical angle.

PROP. 48. Given the base and area of a triangle: to find the locus of the vertex.



Let AB be the given base, and upon it construct the parallelogram $AEDB$ equal to double the given area: the right line ED, produced indefinitely both ways, is the required locus.

For, take any point, C, on ED or ED produced, and join AC, BC. The triangle ABC is (I. 41) half the parallelogram $AEDB$, and therefore equal to the given area.

EXAMPLES.

1. Given two sides of a triangle, and the area: construct it.

2. Given, of a triangle, the base, area, and either of the base angles: construct it.

3. Given, of a triangle, the base, area, and the bisector of the base: construct it.

4. Given the hypotenuse of a right-angled triangle, and the area: construct it. When are there two solutions? When only one? when none?

[Lay down the hypotenuse, and find the locus of the vertex as in the last proposition. The remainder of the solution is evident from Prop. 2.

According as half the hypotenuse is greater than, equal to, or less than, the distance between the parallels, there will be two solutions, one solution, and no solution.]

PROP. 49. Given the base of a triangle, and that the difference of the squares of the sides is equal to a given square: to find the locus of the vertex.

Divide BC, the given base, in H*, so that $BH^2 - CH^2$ shall be equal to the given difference of the squares of the sides (42). Erect HL perpendicular to BC, and produce it indefinitely: HL is the required locus.

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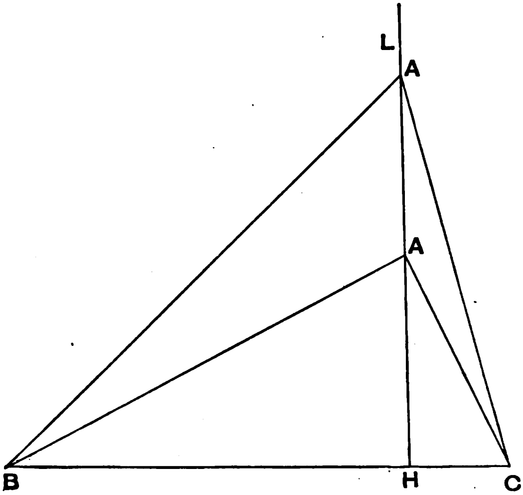
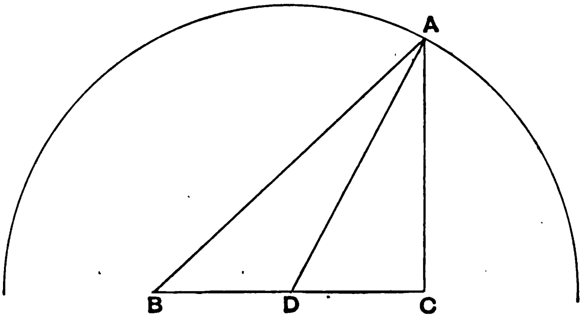
* The point H will be on the production of the base when a side of the given square is taken > the given base. See Prop. 42.

For, take any point A on HL, and join BA, CA.



Then, since AH is a perpendicular to BC, $BA^2 - CA^2 = BH^2 - CH^2$ (11), that is, equal to the given difference of squares.

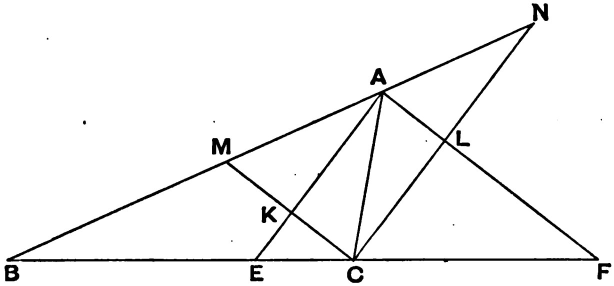
Prop. 50. Given the base of a triangle, and that the sum of the squares of the sides is equal to a given square: to find the locus of the vertex.



Let *BC* be the given base; and suppose that *BAC* is a triangle having the sum of the squares of the sides *BA*, *CA*, equal to a given square. Bisect *BC* in *D*. Join *AD*.

By (12) $BA^2 + CA^2 = 2BD^2 + 2DA^2$; $\therefore 2BD^2 + 2DA^2$ is equal to the given square. But $2BD^2$ is constant, since *BD*, half the fixed base, is fixed; therefore (25) *DA* can be found. The locus of *A* is, then, a circle with centre *D*, and radius *DA* found as above.

PROP. 51. Given the base, and ratio of the sides of a triangle: to find the locus of the vertex.



Let *BC* be the given base. Divide it (VI. 10) internally at *E*, and externally at *F*, so that

$$AE : EC :: AF : FC :: \text{the given ratio.}$$

Upon *EF* describe a semicircle: this is the required locus. Take any point, *A*, on the circumference of this semicircle; and join *BA*, *CA*, *FA*, and *EA*. Draw *CK*, *CL* perpendiculars to *AE*, *AF*, respectively, and meeting *AB* in *M*, *N*, respectively.

Since the angles *CKA*, *CLA*, *KAL* are each right, the angle *KCL* must be a right angle (8), and the lines *CM*, *AF*, as well as the lines *CN*, *AE* are parallel. Hence (VI. 2),

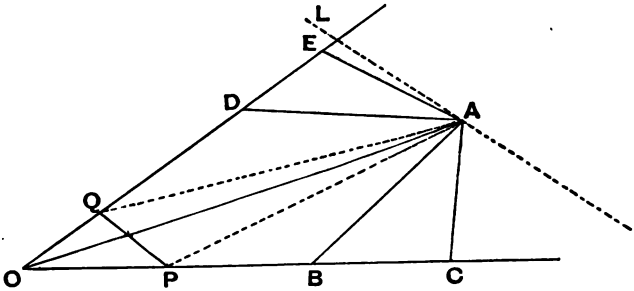
$$BE : EC :: BA : AN, \text{ and}$$
$$BF : FC :: BA : AM; \text{ but}$$
$$BE : EC :: BF : FC, \text{ by construction,}$$
$$\therefore BA : AN :: BA : AM;$$

and, therefore, *AN*, *AM* are equal.

But the angle *MCN* was proved to be a right angle; therefore (2) the three lines *AM*, *AC*, *AN* are equal; but *AK* is at right angles to *CM*; therefore (I. 5, 26) it bisects the angle *BAC*; and therefore (VI. 3)

$$BA : AC :: BE : EC :: \text{the given ratio.}$$

PROP. 52. The bases of two triangles are given in magnitude and position, to find the locus of their common vertex when the sum of their areas is also given.



Let *BC*, *ED* be the two fixed bases of the triangles *ABC*, *EDA*, having a common vertex, *A*, and the sum of their areas constant: it is required to find the locus of the point *A*. Let *DE*, *BC*, produced, meet in *O*. Take off (I. 3) *OQ*, *OP* equal to *ED*, *BC*, respectively. Join *QP*, *QA*, *PA*, and *OA*.

The triangles *DEA*, *OQA* are equal (I. 38). Similarly the triangles *BAC*, *OPA* are equal; therefore the quadrilateral *OPAQ* is equal to the sum of the triangles *ABC*, *DAE*, and therefore of given area. But the triangle *OPQ* is fixed in every respect; therefore the area of the triangle *PAQ* is constant, no matter how the vertex, *A*, moves. The locus, then, of the point *A* is (48) a straight line parallel to the fixed base *PQ*.

Note. The locus is limited to the intercept of the parallel between the angle *COE*. When the common vertex takes a position, *L*, outside this angle, it will be found that the difference between the areas of the triangles is then equal to the quadrilateral *OPAQ*.

EXAMPLES.

1. The bases of three triangles are given in magnitude and position; find the locus of their common vertex when the sum of their areas is also given.

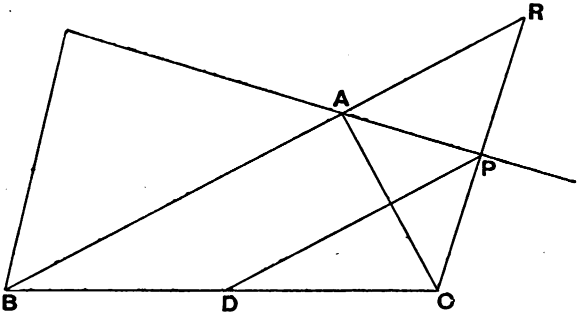
[Leaving the third triangle alone for a moment, it is evident from the last that two of the triangles, *DEA*, *BCA*, can be reduced to the area of a quadrilateral *OQPA*, of which the triangle *POQ* is fixed in every way.

Now taking in the third triangle *HGA*, having *HG* for fixed base, the problem is evidently reduced to find the locus of the common vertex *A*, of two triangles *PAQ*, *HGA*, standing upon two bases *PQ*, *HG*, given in magnitude and position, and the sum of whose areas is constant. The problem is thus reduced to the preceding.]

2. Given three straight lines in position, find the locus of a point the sum of whose distances from the three lines shall be constant.

[By taking any three equal segments on the three straight lines, the problem is reduced to the preceding, at once, by means of Prop. 7.]

Prop. 53. Given the base of a triangle, in magnitude and position, and the sum of the sides: to find the locus of the foot of the perpendicular from either end of the base on the external bisector of the vertical angle.



Let *ABC* be a triangle, of which the base *BC* is given in magnitude and position, and the sum of the sides *BA*, *AC* of constant length: it is required to find the locus of the point, *P*, where *CP* meets *AP*, the external bisector of the angle *A*, at right angles. Join *D*, the middle point of *BC*, and *P*; produce *CP* meeting *BA*, also produced, in *R*.

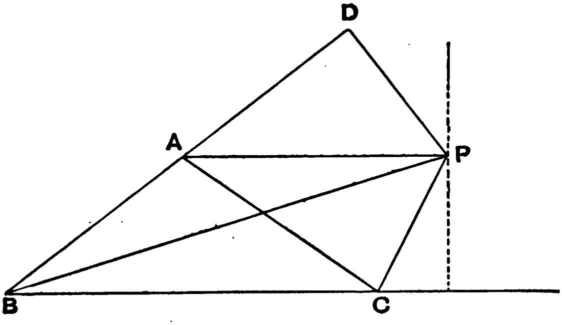
Since the angles at *P* are right, and the angles *CAP*, *PAR* are equal by hypothesis, the triangles *CAP*, *PAR* are (I. 26) equal in every respect; therefore *CP*, *PR* are equal; or *CR* is bisected in *P*. Hence (1) *DP* is half of *BR*. But *BR*, being equal to the sum of *BA*, *AC*, is therefore given in length: hence *DP* is given in length. But *D* is a fixed point; therefore the locus of *P* is a circle, with centre *D*, and a radius *DP* of constant length—half the given sum of the sides.

EXAMPLE.

Given the base of a triangle and the difference of the sides, find the locus of the foot of the perpendicular from either end of the base on the internal bisector of the vertical angle.

[This problem is analogous to the preceding. If *P* be the foot of the perpendicular in this case, and *D* the middle point of the base, it will be found without any difficulty that *DP* is of constant length, that is, equal to half the difference of the sides. The locus is, therefore, a circle.]

PROP. 54. Given the base of a triangle, and the sum of the sides; to find the locus of the point where the perpendicular at either end of the base to the adjacent side meets the external bisector of the vertical angle.



Let *BAC* be a triangle of which the base *BC* is given in magnitude and position, and the sum of the sides *BA*, *AC* also given: it is required to find the locus of the point, *P*, where *CP*, at right angles to *AC*, meets *AP*, the bisector of the angle *CAD*, in *P*.

Join *BP*, and draw *PD* perpendicular to *BA* produced.

In the triangles *PAD*, *PAC*, the side *AP* is common, and the angles *PAD*, *PDA* are equal, respectively, to the angles *PAC*, *PCA*; therefore (I. 26) the lines *PD*, *PC* are equal, and the lines *AD*, *AC* are also equal. Hence *BD* is equal to the given sum of sides. Again (I. 47),

$$BP^2 - PD^2 = BD^2, \text{ or } BP^2 - PC^2 = BD^2;$$

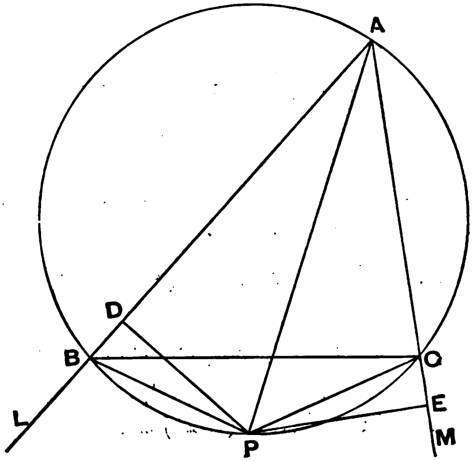
that is, the difference of the squares of *BP*, *PC* is equal to the square on the given sum of sides. But the base *BC* of the triangle *BPC* is fixed, therefore (49) the locus of *P* is a right line.

Example.

Given the base of a triangle and the difference of the sides, find the locus of the point where the perpendicular at either end of the base to the adjacent side meets the internal bisector of the vertical angle.

[This problem is analogous to the preceding. If *P* be the point, in this case, whose locus we require, it will be found that *P* is the vertex of a triangle *BCP*, whose base *BC* is fixed, and the difference of the squares of its sides is equal to a given square—the square on half the given difference of the sides. The problem is thus reduced to Prop. 49.]

Prop. 55. Given, in position, the vertical angle of a



triangle, and the sum of its sides: to find the locus of the centre of the circumscribing circle.

Let *LAM* be the fixed vertical angle; and let *ABC* be one position of a triangle having the sum of its sides *BA*, *AC*, of given length.

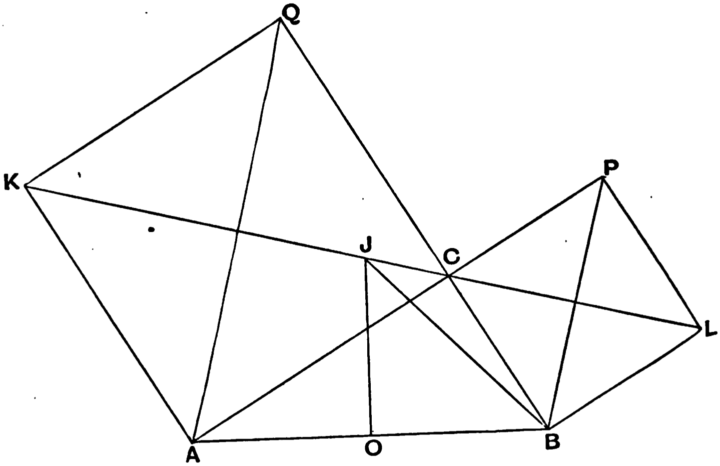
Describe a circle about *ABC*: it is required to find the locus of the centre of this circle.

Bisect the angle *BAC* by *AP*, meeting the circumference in *P*. Draw *PD*, *PE* perpendicular to *AB*, *AC* respectively. Join *BP*, *PC*.

In the triangles *BPD*, *CPE*, the angles *PCE*, *ABP* are (17) equal; the angles *BDP*, *PEC* are also equal, each being right, and the sides *BP*, *PC* equal, since they subtend equal angles at the point *A*; therefore (I. 26) the sides *BD*, *CE* are equal. Again, in the triangles *APD*, *APE*, the side *AP* is common, and the angles *PAD*, *ADP* are equal, respectively, to the two *PAE*, *PEA*; therefore (I. 26) the sides *AD*, *AE* are equal. Hence the sum of the sides *AB*, *AC* is equal to the sum of *AD*, *BD*, *AC*, or the sum of *AD*, *AE*, or double *AD*. Therefore *AD* is given and fixed; and the angle *PAD* being fixed, and *DP* being at right angles to *AD*, the point *P* is, therefore, a fixed point. Hence all circles cutting off portions *AB*, *AC*, whose sum is constant, pass through two fixed points *A*, *P*. The locus, then, of the centres of circles passing through the two points, is clearly (III. 3), the perpendicular at the middle point of the line joining them.

PROP. 56. Given the hypotenuse of a right-angled triangle: to find the loci of the edges of the squares described upon the sides.

Let *ABC* be one position of a right-angled triangle, *C* being the right angle, and having the hypotenuse *AB* fixed.



Describe the squares *BCPL*, *ACQK*: it is required to find the loci of the different points *L*, *P*, *Q*, *K*.

Join *BP*, *AQ*.

Since *BC*, *CP* are equal, and the angle *BCP* a right angle, the angle *CPB* is (I. 32) half a right angle. Similarly the angle *AQB* is half a right angle. Hence, in the triangles *APB*, *AQB*,* there is a common fixed base, *AB*, and also an equal given vertical angle, therefore the locus of their vertices is a segment upon *AB*, and containing an angle equal to half a right angle (45).

To find the loci of the points *L*, *K*.

Join *KC*, *CL*. It is necessary first to prove that *KCL* is a straight line. Since the angles *ACK*, *LCB* are each half a right angle, and the angle ACB is a right angle, the line AC makes with the lines KC, CL, angles, the sum of which is equal to two right angles; therefore (I. 14) the line KCL is a straight one.

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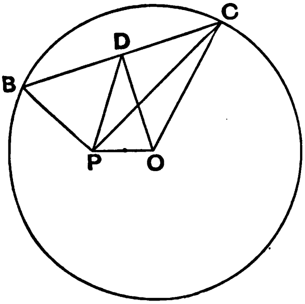
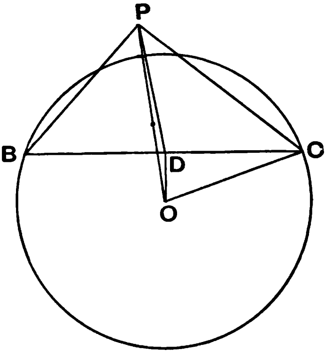
* The lines *AC*, *CP*, form (I. 14) one straight line, since the angles *ACB*, *BCP*, are each right. Similarly *BCQ* is one straight line.

Through O, the middle point of AB, erect a perpendicular OJ, meeting KL in J. Then (29) J is a fixed point; therefore the line BJ is fixed. But the angle BLJ is half a right angle; therefore the locus of L is a segment of a circle upon BJ, containing an angle equal to half a right angle. Similarly the locus of K is a segment of a circle upon AJ, and containing an angle, also, equal to half a right angle.

Prop. 57. A chord of a given circle subtends a right angle at a fixed point: to find the locus of its middle point.





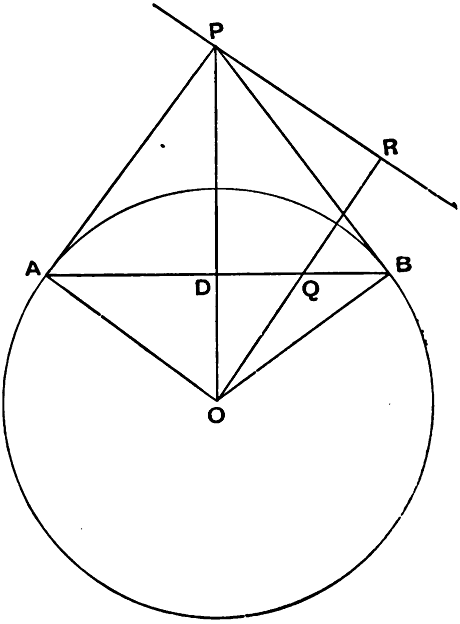
Let P be the fixed point, and O the centre of the given circle: it is required to find the locus of the middle points of all chords, BC, subtending a right angle at P.

Join O to D, the middle point of one such chord BC. Join OP.

Since the angle BPC is right, and D is the middle point of BC; BD, DC, PD are all equal (2).

And the angle ODC is right (III. 3); therefore (I. 47) $OC^2 = OD^2 + DC^2 = OD^2 + DP^2$. But OC, being the radius of the circle, is always constant; therefore $OD^2 + DP^2$ is constant. Hence, of the triangle PDO, its base OP is fixed, and the sum of the squares of its sides is given; therefore (50) the locus of the vertex D is a circle with the middle point of OP as centre, and a radius of known length.

Prop. 58. A chord of a given circle passes through a fixed point: to find the locus of the intersection of the tangents at its extremities.



Let AB be any chord of the circle (O), passing through the fixed point Q; let the tangents at A, B intersect in P: it is required to find the locus of the point P.

Join OP, cutting AB in D. Join OA, OB; and draw PR perpendicular to OQ produced.

It is necessary, first, to prove that the tangents PA, PB are equal, and that OP is at right angles to AB.

Since the angles PAO, PBO are each right (III. 18), $OP^2 = OA^2 + AP^2 = OB^2 + BP^2$. But OA, OB being equal, their squares are equal, therefore the squares on AP, BP are equal, and therefore the lines themselves are equal.

Hence, in the triangles OAP, OBP, the sides OA, AP are equal to the sides OB, BP, and the base OP common; therefore the angles OPA, OPB are equal. Hence, again, in the triangles PAD, PBD the two sides AP, PD are equal to the two BP, PD, and the contained angles APD, BPD equal; therefore (I. 4) the angles ADP, BDP are equal. Hence these angles are each right.

Again, in the triangle OAP, the angle OAP is right, and AD is a perpendicular; therefore (32) $OP \,.\, OD = OA^2$. But, since the angles PDQ, PRQ are each right, $OP \,.\, OD = OQ \,.\, OR$ (17). Hence $OQ \,.\, OR = OA^2$.. That is, $OQ \,.\, OR$ is constant. But OQ is fixed, therefore OR is given and fixed. But RP is at right angles to OR; therefore RP is fixed, and is the required locus.

PROP. 59. To find the locus of the intersection of equal tangents to two given circles.

Let A, B be the centres of (fig. 1) two non-intersecting circles; and suppose that the tangents PL, PM are equal: it is required to find the locus of this point P.

Join AL, BM, AP, BP. Draw PD perpendicular to AB.

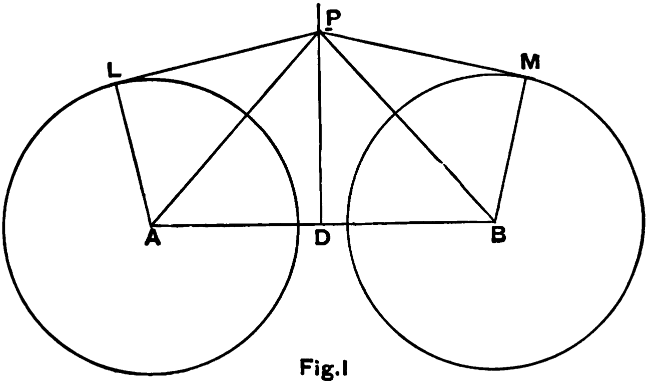


Fig. 1

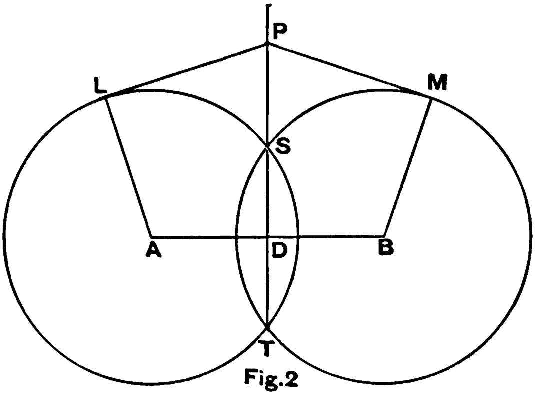


Fig. 2

Since the angle ALP is right, $PL^2 = AP^2 - AL^2$. Similarly $PM^2 = BP^2 - BM^2$. But PL^2, PM^2 are equal by hypothesis; therefore

$$AP^2 - AL^2 = BP^2 - BM^2,$$

or $$AP^2 - BP^2 = AL^2 - BM^2,$$

or $$AD^2 - BD^2 = AL^2 - BM^2 \quad (11).$$

This shows that the point, P, whose locus we are seeking, lies on a straight line perpendicular to the line of centres, and dividing this line, AB, into segments, the difference of the squares of which $AD^2 - BD^2$ is equal to the difference of the squares of the radii $AL^2 - BM^2$. Hence, to get the locus, we have only to divide AB in D, so that (42) $AD^2 - BD^2$ shall be equal to the difference of two given squares AL^2, BM^2, and to erect a perpendicular at this point, D. It will be readily found, then, that if from *any* point, P, on this straight line, tangents, PL, PM, be drawn to the circles (A), (B), they are equal.

In the case, (fig. 2), where the circles intersect, it is clear that ST, the common chord of the circles, produced, is the required locus.

For, taking any point, P, on the production of ST, and drawing tangents PL, PM, to the circles (A), (B), we have (III. 36) $PS \cdot PT = PL^2 = PM^2$, that is, the tangents PL, PM are equal.

In this case, as in the other, it will be found that the line PST divides AB at right angles, and into segments the difference of the squares of which is equal to the difference of the squares of the radii.

EXAMPLE.

Find the point, the tangents drawn from which to three given circles shall be equal.

[First, find the locus of the intersection of equal tangents to any two of the circles by the preceding: then, find the locus of the intersection of equal tangents to any other two of the circles. The intersection of the two loci is clearly the required point.

If the centres of the circles were in a straight line, the point would then be at infinity.]

PROP. 60. From a fixed point, P, a straight line PQ is drawn meeting a fixed indefinite straight line, LM, in Q: to find the locus of a point, R, (1) where $PR \,.\, PQ$ is equal to a given square; (2) where $PR : PQ$ in a given ratio.

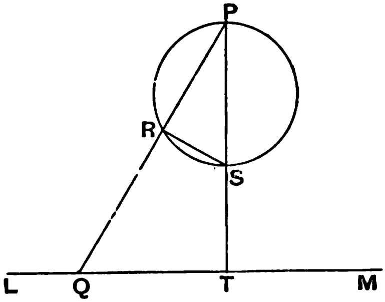
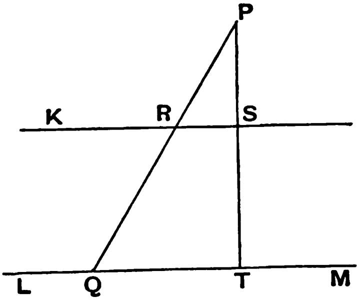


Fig. I



(1) Let (fig. 1) PQ be any straight line divided in R,

so that $PR.PQ$ is equal to a given square: it is required to find the locus of the point R.

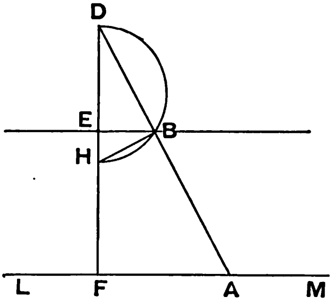
Draw PT perpendicular to the fixed line LM. At R erect RS perpendicular to PQ, meeting PT in S.

Since the angles QRS, QTS are each right, by construction, $PR.PQ = PS.PT$ (17); therefore $PS.PT$ is equal to a given square. But PT is fixed; therefore PS is determined and fixed: and the angle PRS is a right angle; therefore the locus of the point R is a circle upon PS as diameter.

(2) To find the locus of the point R, in the second case. Draw PT perpendicular to LM. Divide PT in S, so that $PS:PT$ in the given ratio (VI. 10). Draw SK parallel to LM, and produce it indefinitely: this is the required locus. For, evidently (VI. 4), every straight line PQ, meeting SK in R, is divided, so that

$$PR : PQ :: PS : PT :: \text{the given ratio.}$$

PROP. 61. Given the rectangle under two lines, and their ratio: to find the lines.



Let any straight line DF be divided in E, so that

$DF : DE$ in the given ratio. If $DF . DE$ is equal to the given rectangle, the problem is solved. If not, take DH, so that $DF . DH$ is equal to the given rectangle. At F erect LFM perpendicular to DF. Upon DH describe a semicircle. Through the point E draw a straight line parallel to LFM, and cutting the semicircle in B. Join DB, and produce it till it meets LFM in A:

DA, DB are the required lines. For (VI. 4)

$$DA : DB :: DF : DE,$$

that is, in the given ratio; and (17) $DA . DB = DF . DH$, the given rectangle, since the angles DFA, DBH are, clearly, each right.

EXAMPLE.

Given, of a triangle, the base, the rectangle contained by the sides, and the ratio of the sides: construct the triangle.

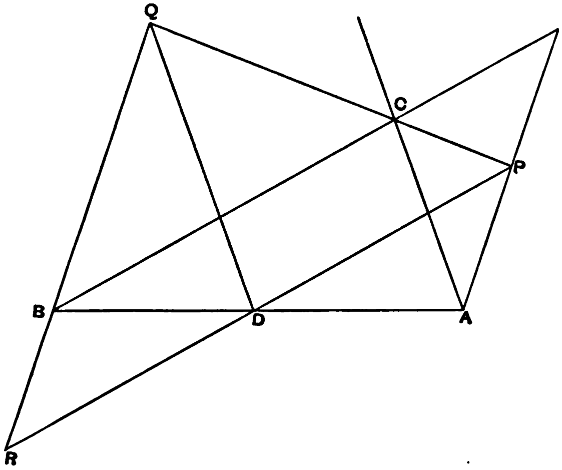
[By means of the preceding the problem is reduced to (I. 22).]

PROP. 62. Given, of a triangle, the base and sum of the sides, to prove that, in any position of the vertex, the rectangle under the perpendiculars drawn from the ends of the base to the external bisector of the vertical angle is constant, and equal to the difference of the squares of half the sum of the sides and of half the base.

Let ABC be any triangle, of which the base AB is given and fixed, and the sum of the sides BC, AC constant. From A, B draw the perpendiculars AP, BQ to the external bisector of the angle C: then $AP . BQ$ is constant and equal to

$$\left(\frac{BC + CA}{2}\right)^2 - \left(\frac{AB}{2}\right)^2.$$

Join D, the middle point of AB, to Q, P; and let PD, produced, meet QB, produced, in R.



Since QR, AP are parallel, the angles BRD, APD are equal, as are, also, the angles RBD, PAD (I. 29); and the sides BD, DA are equal in the two triangles BDR, APD; therefore (I. 26) the sides BR, AP and DP, DR are equal; and, therefore (2), the three DQ, DP, DR are equal. Hence the triangle QDR is isosceles, and, therefore (30)

$$BQ \, . \, BR = DR^2 - DB^2;$$

but BR has been proved equal to AP, and DR to DP; therefore

$$BQ \, . \, AP = DP^2 - DB^2;$$

but DP is half the sum of the sides BC, AC (see proof, Prop. 53), and DB is half the base, hence, finally,

$$BQ \, . \, AP = \left(\frac{BC + CA}{2}\right)^2 - \left(\frac{AB}{2}\right)^2.$$

EXAMPLE.

Given, of a triangle, the base and difference of the sides, to prove that, in any position of the vertex, the rectangle under the perpendiculars drawn from the ends of the base to the internal bisector of the vertical angle is constant, and equal to the difference of the squares of half the base and of half the difference of the sides.

[This is the analagous theorem to the preceding. If *AP*, *BQ* be the perpendiculars, and *D* the middle point of *AB*, then, if *DP*, produced, meet *BQ* in *R*, it will be found that the triangle *DRQ* is isosceles, and that

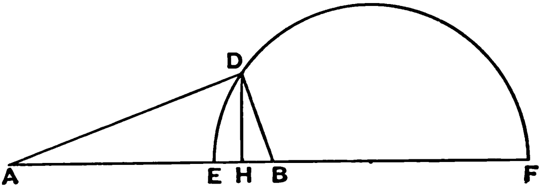
$$BD^2 - DR^2 = BQ \, . \, BR,$$

that is,

$$BQ \, . \, AP = \left(\frac{AB}{2}\right)^2 - \left(\frac{BC-CA}{2}\right)^2.$$

(See Example to Prop. 53)].

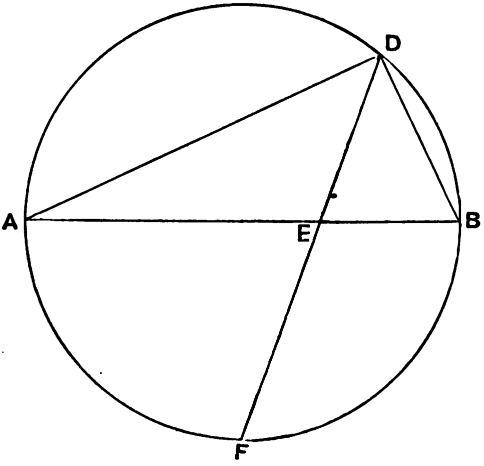
PROP. 63. Given the difference of the squares of two lines, and their ratio: to find the lines.



Take any straight line *AB*, and divide it internally in *E*, and externally in *F*, so that *AE* : *EB* :: *AF* : *FB*, so as the given ratio (VI. 10). If $AE^2 - EB^2$ is equal to the given difference of the squares, the problem is solved.

If not, divide *AB* in *H*, so that (42) $AH^2 - BH^2$ is equal to the given difference of the squares. At *H*, erect *HD* perpendicular to *AB*, and meeting the semicircle described upon *EF* as diameter in *D*. Join *AD*, *BD*. These are the required lines. For (11), they satisfy the condition that $AD^2 - BD^2 = AH^2 - BH^2$, the given difference of the squares; and (51) $AD : BD :: AE : BE$, the given ratio; since D is a point both on the perpendicular DH, and on the semicircle EDF.

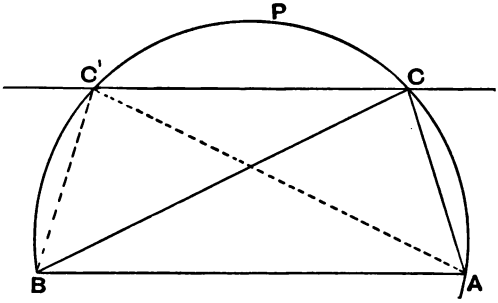
PROP. 64. Given the sum of the squares of two lines, and their ratio: to find the lines.



Take AB equal to a side of the square that is equal to the given sum of the squares of the lines we are seeking. Upon AB, as diameter, describe a circle. Bisect the arc AFB in F. Divide AB in E, so that $AE : EB$ in the given ratio. Join FE, and produce FE to meet the circumference in D. Join AD, BD: these are the required lines.

For, since the arcs AF, BF are equal, the angles ADF, BDF are equal, and therefore (VI. 3) $AD : BD :: AE : BE$, that is, in the given ratio; and (I. 47) the sum of their squares is equal to the given square on *AB*.

Prop. 65. Given, of a triangle, the base, vertical angle, and area: to construct it.



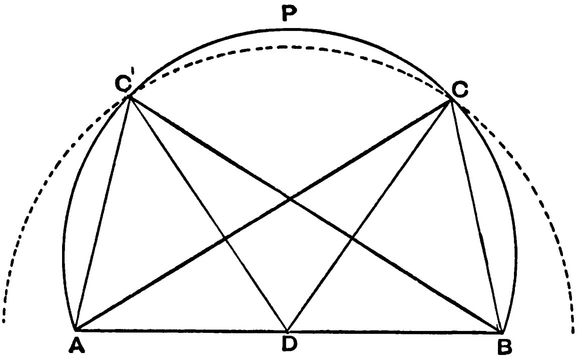
Let *AB* be the given base, and upon it describe the segment *BPA*, containing an angle equal to the given vertical angle.

Since the area of the triangle is given, its vertex must lie on a parallel to the base (48). Let this parallel cut the circumference of the circle in *C*, *C'*. Join *BC*, *AC*, *BC'*, *AC'*. Either of the triangles *ABC*, *ABC'* satisfies the conditions of the problem.

For, of either, the vertical angle is given, since its vertex lies on the arc *BPA*; and its area is given, since its vertex is on the parallel *CC'*.

Note. There are, in general, two solutions to the above problem. For every right line, in general, cuts a circle in two points. There is only one solution when the parallel *CC'* touches the circle; and no solution when it neither cuts nor touches it.

PROP. 66. Given, of a triangle, the base, vertical angle, and sum of the squares of the sides: to construct it.

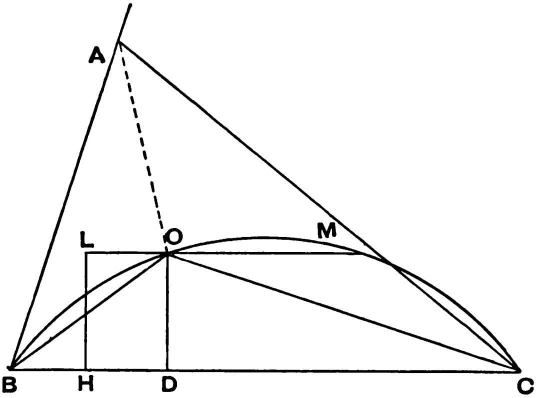


Let AB be the given base. Upon AB describe (III. 33) the segment APB, capable of containing an angle equal to the given vertical angle. Since the sum of the squares of the sides is given, the locus of the vertex is a circle with the middle point of AB as centre, and a radius of known length (50). Let this circle intersect the segment APB in C, C'. Join AC, BC, AC', BC'. Either of the triangles ABC, ABC' satisfies the conditions of the problem.

For, the vertical angle of either of them is given, since its vertex lies on the arc APB; and the sum of the squares of the sides is given, since its vertex lies on the circumference of the circle, whose centre is D, and radius DC or DC'.

PROP. 67. Given, of a triangle, the base, vertical angle, and radius of the inscribed circle: to construct it.

Upon the given base BC describe a segment of a circle containing an angle equal to the sum of a right angle and half the given vertical angle (see Prop. 46, *b*). At any point, *H*, in the base, erect *HL* perpendicular, and equal to the given radius of the inscribed circle. Through the point *L*, draw the straight line *LOM* parallel to the base, and cutting the segment of the circle described upon *BC* in *O*, *M*. From either point, *O*, draw *OD* perpendicular to *BC*. Join *BO*, *CO*; and with these lines make the angles *OBA*, *OCA* equal to the angles *OBD*, *OCD*, respectively. The triangle *BAC*, thus formed, is the required one.



Join *OA*. Since the bisectors of the angles of a triangle meet in a point (22), and the angles *B*, *C*, are bisected by construction, *O* is the centre of the inscribed circle. Hence (46, *b*) the angle *BOC* is equal to the sum of a right angle and the angle *A*. But the angle *BOC*, by construction, is equal to the sum of a right angle and the given vertical angle; therefore the angle *A* must be the given vertical angle. And clearly *OD* is equal to *LH*, the given radius.

EXAMPLES.

1. Find the locus of the middle points of parallel chords in a given circle.

[Consider (III. 3).]

2. Draw a chord in a given circle, so that it shall subtend a right angle at a fixed point, and be parallel to a given straight line.

[The locus of the middle points of all chords parallel to a given straight line being determined by the preceding, and the locus of the middle points of chords subtending a right angle at a fixed point being determined by Prop. 57, *the* middle point of a chord, satisfying both conditions, is therefore determined, and hence the chord itself.]

3. Given the base, vertical angle, and difference of the squares of the sides: construct the triangle. When are there two solutions? When is there no solution?

[Consider Props. 45, 49.]

4. Given the base, difference of the squares of the sides, and sum of the squares of the sides: construct the triangle.

[This problem may be solved by means of two loci (see Props. 49, 50); or, the sides may be actually determined by means of Prop. 43, when the problem is clearly reduced to (I. 22.)]

5. Given the base, area, and difference of the squares of the sides: construct the triangle.

6. Given the base, area, and sum of the squares of the sides: construct the triangle.

[Consider Props. 48, 50.]

7. Given the base, vertical angle, and the ratio of the sides: construct the triangle.

[This problem can, of course, be solved by means of the two loci determined in Props. 45, 51; but the solution is better obtained in the same manner as in Prop. 64, which latter is clearly a particular case of the present example.]

8. Given the base, area, and the ratio of the sides of a triangle: construct it. When is there only one solution?

9. Given the base, difference of the squares of the sides, and ratio of the sides of a triangle: construct it. When is there no solution?

[Consider Props. 49, 51.]

10. Given the base, sum of the squares of the sides, and ratio of the sides: construct the triangle.

11. Given, of a triangle, the base, vertical angle, and radius of any one of the escribed circles: construct it.

[This problem is solved in a similar manner to Prop. 67.]

ON MAXIMA AND MINIMA.

Def. *Of all straight lines, angles, or areas, subject to given conditions, the maximum is that which is the greatest, the minimum is that which is the least.

Prop. 68. To divide a given straight line into two parts, so that the sum of the squares of the segments (1) shall be given: (2) shall be a minimum.

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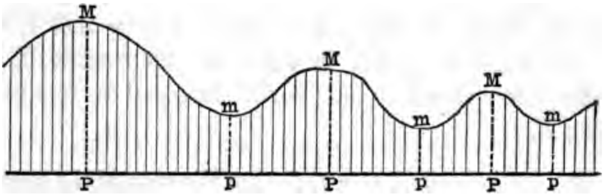
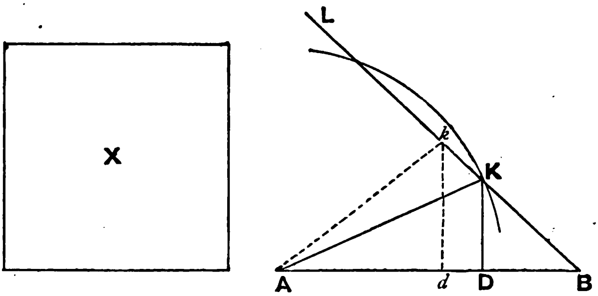
* Though the above definition may suit as far as the point, line, and circle are concerned, still the following is a more general one:—

When any variable magnitude, linear or otherwise, subject to given conditions, passes gradually from an increasing to a decreasing state, or vice-versa, it has, in the former case, gone through a maximum value, and in the latter through a minimum.



The dotted lines *MP*, *mp*, in the diagram, are, respectively, maxima and minima.

Let AB be the given straight line: it is required (1) to divide it into segments, the sum of the squares of which shall be equal to a given square X.



At B, with the straight line AB, make the angle ABL equal to half a right angle. With A as centre, and a radius equal to a side of the square X, describe a circle. Let this circle cut the line BL in K; join AK; and draw KD perpendicular to AB: AD, BD are the required segments.

In the triangle BDK, the angle B is half a right angle, and the angle D is a right angle; therefore the angle DKB is half a right angle (I. 32); and, therefore, the sides DK, DB are equal (I. 6). Again, since the angle ADK is right, (I. 47) $AK^2 = AD^2 + DK^2 = AD^2 + BD^2$. But the square on AK is equal to the square, X, by construction; therefore $AD^2 + BD^2$ is equal to the given square X.

(2) *To divide AB in D so that the sum of the squares of AD, BD shall be a minimum.

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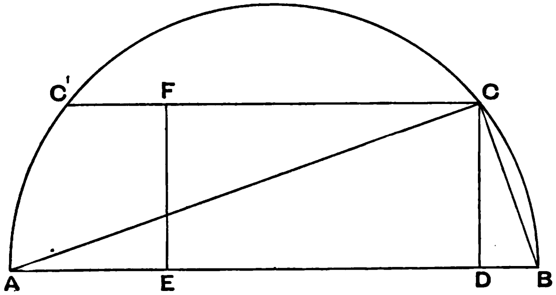
* The student can solve (2), independently, by means of (II. 9).

The construction being the same, we see that the sum of the squares of any two segments, *AD*, *BD*, varies as the square of a line drawn from *A* to the line *BL*. Hence, when the sum of the squares of *AD*, *BD* is a minimum, the line *AK* is a minimum, which is (I. 19) the case when *AK* is perpendicular to *BL*. The point, *D*, will then become (I. 32, 26) the bisection of *AB*, that is to say, the segments are equal.

PROP. 69. To divide a given straight line into two parts, so that the rectangle under the segments (1) shall be given: (2) shall be a maximum.

Let *AB* be the given straight line: it is required (1) to divide it into segments, the rectangle under which shall be equal to a given square.

Upon *AB* describe a semicircle, and at any point, *E*, in the base, erect *EF* perpendicular, and equal to a side of the given square. Through *F* draw *FCC′* parallel to *AB*, and cutting the circumference of the semicircle in *C*, *C′*; draw *CD* perpendicular to *AB*: *AD*, *BD* are the required segments.



For, since the angle ACB is right, being in a semicircle, $AD \,.\, BD = CD^2$ (10); but CD, FE are equal, being the opposite sides of a rectangle; therefore

$$AD \,.\, BD = FE^2 = \text{the given square.}$$

(2) *To divide AB in D so that $AD \,.\, BD$ shall be a maximum.

From the first part, we see that $AD \,.\, DB$ varies as the square of CD. Hence, the rectangle will be a maximum when the square is a maximum; that is, when CD is a maximum; which, clearly, is the case when C is the bisection of the semicircular arc. The point, D, is, then, obviously, the bisection of the line AB.

Note. (1) may evidently be stated thus:—

Given the rectangle under two lines, and the sum of the lines: to find them.

(2) clearly shews that:—

Of all rectangles of a given perimeter, the square has the maximum area.

EXAMPLE.

Through a fixed point, within or without a given circle, draw a straight line cutting the circle, so that the sum of the intercepted segments shall be equal to a given length. When would the problem be impossible?

[Since the rectangle under the segments of the secant is always constant (III. 35, 36), according as the point is within or without the circle, the problem is therefore reduced to the above. See Note (1).]

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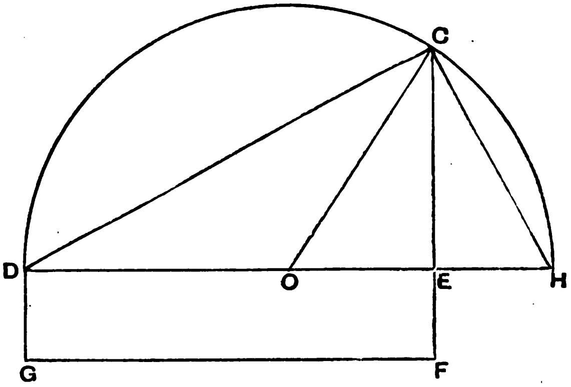
* This, also, can be solved, independently, by means of (II. 5).

Prop. 70. Of all rectangles of a given area, the square has the minimum perimeter.



Let $DEFG$ be any rectangle of the given area. Produce DE to H, so that EH is equal to EF. Upon DH, as diameter, describe a semicircle. Let EF, produced, meet the semicircle in C. Join DC, HC, OC, O being the middle point of DH.

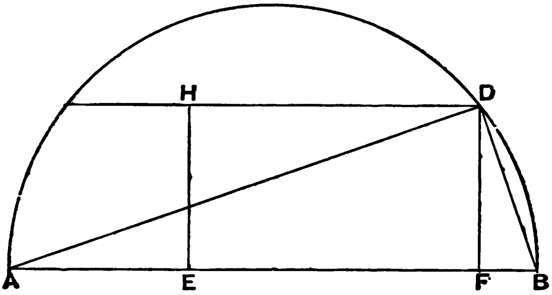
Since the angle DCH is right, being in a semicircle, and CE is perpendicular to DH, then (10) $DE . EH = CE^2$. But EH, EF are equal, therefore the rectangle $DEFG$ is equal to the square on EC.

Now, the perimeter of the rectangle is, obviously, double the line DH, or four times the line OC. But, in the right-angled triangle OCE, the hypotenuse OC is greater than the side CE (I. 19); therefore four times the line OC is greater than four times the line CE, in other words, the perimeter of the rectangle is greater than the perimeter of the square.

PROP. 71. Given that the sum of the squares of two lines is equal to a given square: to find the lines,

(1) When the rectangle under them is given.

(2) When the rectangle under them is a maximum.



(1) Let AB be a side of the given square; and upon it describe a semicircle.

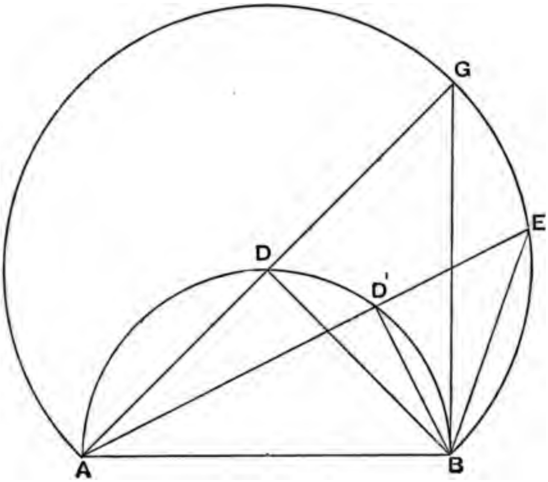
The solution of (1) is, then, reduced to find a point D in the semicircle, so that $AD \,.\, BD$ shall be equal to a given rectangle.

At any point E, then, in the base AB, erect a perpendicular EH, and so taken that $AB \,.\, EH$ is equal to the given rectangle. Through H draw the parallel HD, meeting the semicircle in D. Join AD, DB, and draw DF perpendicular to AB.

Since ADB is a right-angled triangle, and DF is perpendicular to the hypotenuse, $AD \,.\, BD = AB \,.\, DF$ (see Ex. 15, page 11); therefore $AD \,.\, BD$ is equal to the given rectangle, for DF, EH are equal, being the opposite sides of a rectangle; and (I. 47) $AD^2 + BD^2 = AB^2$, the given square; hence AD, BD are the required lines.

To solve (2). From the last, we see that $AD \cdot DB$ varies as the square of the perpendicular DF; hence $AD \cdot DB$ will be a maximum when DF is a maximum, which clearly is the case when D is the point of bisection of the semicircle. The lines AD, DB are, then, (III. 29) equal.

Prop. 72. Given that the sum of the squares of two lines is equal to a given square: to find the lines when their sum is a maximum.



Let AB be a side of the square that is equal to the given sum of the squares of the two lines we are seeking.

Upon AB describe a semicircle.

The problem is, then, reduced to find a point, D, in this circumference, so that the sum of AD, BD shall be a maximum; for, always, (I. 47) $AD^2 + BD^2 = AB^2$, since the angle in a semicircle is right.

Let D' be any point on the circumference. Join AD', BD', and produce AD' to E, so that $D'E$, $D'B$ are equal. Join EB.

Since $D'E$, $D'B$ are equal, the angles $D'EB$, $D'BE$ are equal; and therefore the angle $AD'B$ is double the angle $D'EB$, that is, the latter angle is half a right angle. Hence the point E moves on the arc of a segment of a circle upon AB as base, and containing an angle equal to half a right angle. The problem is, then, further reduced to find the maximum chord, AE, in this circle. This, clearly, (III. 15) is the diameter.

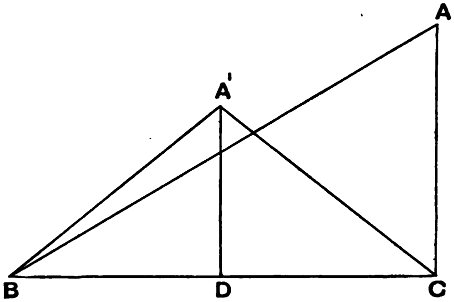
Hence, draw from A the diameter AG, cutting the semicircle in D. Join BD.

AD, BD are the required lines; and obviously D is the centre of the larger circle, and therefore the bisection of the semicircle, whence the lines AD, BD are equal.

PROP. 73. Given in length the two sides of a triangle: to find at what angle they must be placed, so that the area may be a maximum.

Let BC, CA be the two sides.

Let BCA be a position of the triangle when the sides



are at right angles; and BCA' when the sides are at any angle: the triangle BCA is greater, in area, than that of the triangle BCA'.

Draw $A'D$ perpendicular to BC.

Since (7) the area of a triangle is equal the rectangle contained by half the base and altitude, the areas of the triangles BCA, BCA' are, respectively,

$$\tfrac{1}{2}BC \,.\, CA, \quad \tfrac{1}{2}BC \,.\, A'D.$$

But $A'D$ is less than the hypotenuse $A'C$ of the triangle $A'DC$, and therefore less than its equal AC. Hence $\tfrac{1}{2}BC \,.\, CA > \tfrac{1}{2}BC \,.\, A'D$, that is, the area of the triangle ABC is greater than that of any other triangle BCA', each triangle having the same sides, but the former having them at right angles.

EXAMPLES.

1. Given in length the two diagonals of a quadrilateral, at what angle must they be placed so that the area of the quadrilateral shall be a maximum?

2. Shew that, when the quadrilateral is in a position that its area is a maximum, it is immaterial where the diagonals intersect.

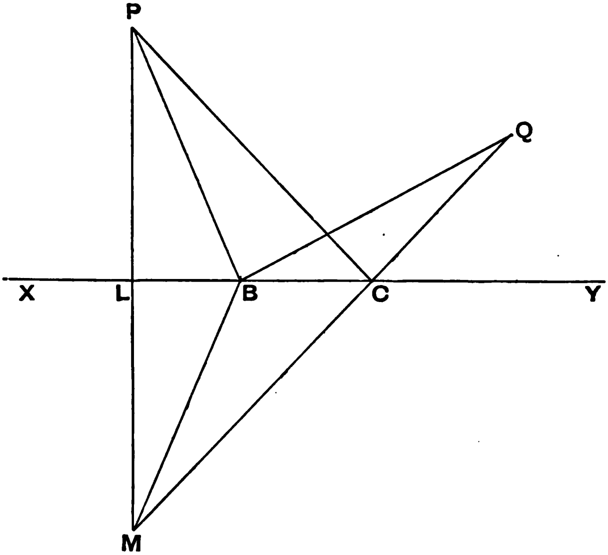
[Consider Prop. 7.]

PROP. 74. To find a point on an indefinite straight line the sum of its distances from two fixed points, lying on the same side of the line, shall be a minimum.

Let XY be the indefinite straight line: it is required to find a point, C, in this line the sum of its distances CP, CQ, from two fixed points P, Q shall be the least possible.

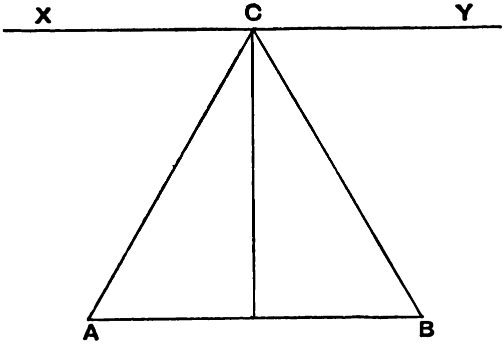
Draw PL perpendicular to XY, and produce it, so that LM is equal to PL. Join MQ, cutting XY in C; this is the required point.

Join *CP*, and take any other point, *B*, in *XY*. Join *PB*, *QB*, and *MB*. In the triangles *PLB*, *MLB*, the sides *PL*, *LB* are equal, respectively, to the sides *ML*, *LB*, and the contained angles *PLB*, *MLB* are also equal, each being right; therefore (I. 4) *PB*, *BM* are equal. Similarly *PC*, *CM* are equal. Again, (I. 20) the sum of *MB*, *BQ* is greater than *MQ*, that is, the sum of *PB*, *BQ* is greater than the sum of *PC*, *CQ*; for *PB*, *MB* have been proved equal, and also *PC*, *CM*.



Note. It is most important to remember that the lines *PC*, *CQ*, as above, whose sum is a minimum, are equally inclined to the indefinite line *XY*. For, (I. 4) the angles *PCL*, *MCL* being equal, and the angles *MCL*, *QCY* being also equal (I. 15), it follows that the angles *PCL*, *QCY* are equal.

PROP. 75. Of all triangles upon the same base and having the same altitude, the isosceles has the minimum perimeter.



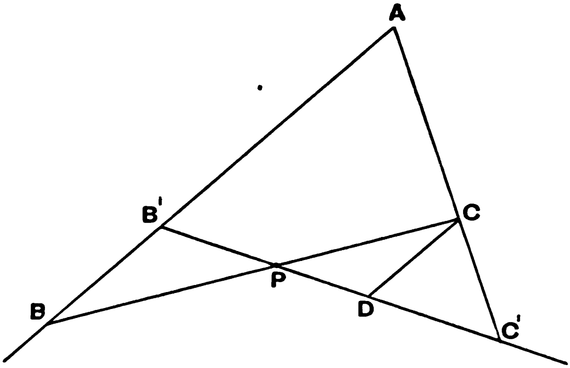
Let ABC be an isosceles triangle, having the angles BAC, ABC equal; through C draw CXY parallel to AB: of all triangles having AB as base, and whose vertices lie upon the line XY, ABC has the minimum perimeter.

For (I. 29) the angles XCA, YCB being equal to the equal angles CAB, CBA, they themselves are equal; and therefore the lines AC, BC are equally inclined to the indefinite line XY. Hence (74, *Note*) the sum of the lines AC, BC is a minimum. This, of course, clearly proves that the perimeter of the triangle ABC is a minimum, for the base AB is constant for every triangle satisfying the conditions.

PROP. 76. Of all triangles having the same vertical angle, and whose bases pass through a fixed point situated within the angle, that, whose base is bisected at the point, has the minimum area.

Let ABC be a triangle, whose base BC is bisected in the point P: its area is less than that of any other triangle $AB'C'$, whose base also passes through P.

Draw CD parallel to AB, meeting $B'C'$ in D.



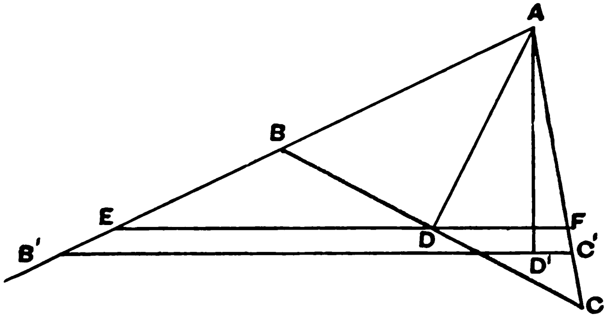
In the triangles $BB'P$, CPD, the sides BP, PC are, equal by hypothesis, and the angles $BB'P$, $B'BP$ are equal respectively, to the angles CDP, PCD (I. 29); therefore (I. 26) the triangles $BB'P$, CPD are equal in area. To each of these equal triangles add the quadrilateral $AB'PC$, whence the triangle ABC is equal to the quadrilateral $AB'DC$; therefore the whole triangle $AB'C'$ is greater than the triangle ABC.

PROP. 77. Of all triangles having the same vertical angle and the same altitude, the isosceles has the minimum area.

Let ABC be an isosceles triangle of vertical angle A, and altitude AD: of all triangles having this vertical angle and altitude, ABC has the minimum area.

Let $AB'C'$ be any other triangle having the same vertical angle A, and an equal altitude AD'. Through D draw DEF parallel to $B'C'$, and meeting AB, AC in E, F, respectively.

Since ABC is an isosceles triangle, and AD is drawn



perpendicular to its base, it (I. 26) bisects the base in D. Hence (76) the area of the triangle ABC is less than the area of EAF, and therefore, obviously, much less than the area of $AB'C'$.

EXAMPLES.

1. Prove that the portion of a variable tangent intercepted between two fixed tangents to a given circle subtends a constant angle at the centre.

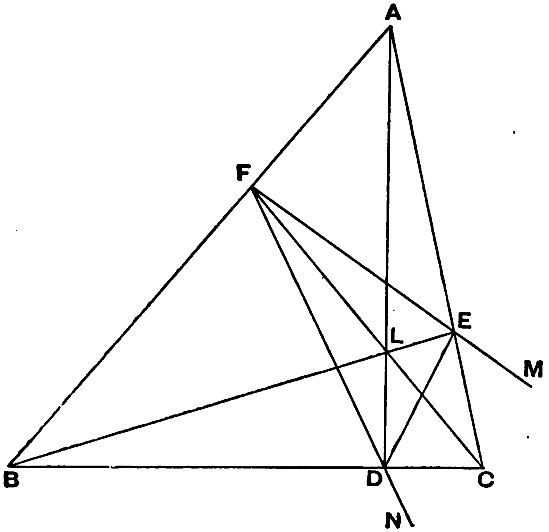
2. Draw a tangent to a given circle so that the triangle formed by it with two fixed tangents shall be of maximum area.

[Since the intercept of the variable tangent subtends a constant angle at the centre of the circle, the triangle, formed by it as base, and the centre as vertex, will be a minimum when the triangle is isosceles (77), for the altitude of the triangle is constant, being the radius of the circle. It will be found, then, that when this triangle is a minimum, the upper is a maximum. In this case, the point of contact of the variable tangent will be the internal bisection of the arc formed by the fixed tangents.]

3. Draw a tangent to a given circle so that the triangle formed by it with two fixed tangents shall be of minimum area.

[In this case the point of contact of the variable tangent will be the external bisection of the arc formed by the fixed tangents.]

PROP. 78. Of all triangles inscribed in a given triangle, that, which has the minimum perimeter, is the triangle formed by joining the feet of the perpendiculars.

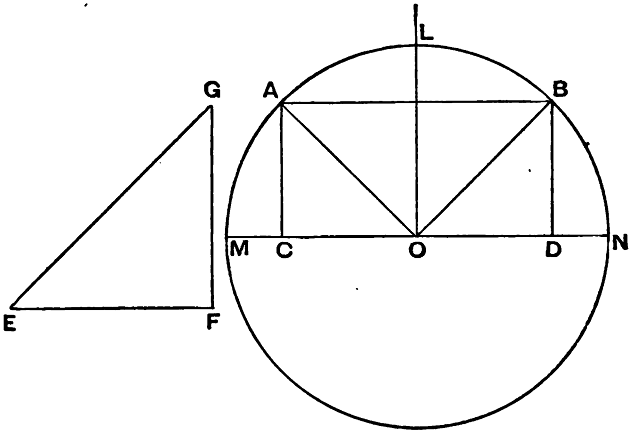


First, suppose *DEF* any triangle inscribed in the given triangle *ABC*. The conditions, then, necessary, that the perimeter of the triangle *DEF* should be a minimum is (74, *Note*) that the angles *EDC, FDB* should be equal, as well as the angles *DFB, EFA*, and *FEA, DEC*. Hence, then, the triangle, whose perimeter is a minimum, must have its sides equally inclined to those of *ABC*. Now let *DEF* be such a triangle having the angles *EDC, FDB*, etc., etc., equal. Bisect the angles *EDF, DEF*, and let the bisectors *DL, EL* meet in *L*. Produce *FE, FD* indefinitely to *M, N*. Since the angles *DEC, AEF* are equal by supposition, and the angles *AEF, CEM* (I. 15), the angles *DEC, CEM* are equal. Hence *EC* bisects the angle *DEM*. Similarly *CD* bisects the angle *EDN*. Hence the lines *CD*, *CE* are two external bisectors of the angles *DEF*, *EDF* of the triangle *DEF*. But (23) the two external and the third internal bisectors of the angles of a triangle meet in a point; therefore the bisector of the angle *EFD* must pass through *C*. But this bisector is obviously perpendicular to the side *AB*, since the angles *EFA*, *DFB* are equal; hence *F* is the foot of the perpendicular from *C* to the opposite side. Similarly *E*, *D* are the other two feet, wherefore the triangle *DEF* is that formed by joining the feet of the perpendiculars.

PROP. 79. To inscribe in a semicircle a rectangle (1) of a given area: (2) of a maximum area.

To solve (1):

Suppose that *ABDC* is the rectangle of given area inscribed in the semicircle *MLN*. Through *O*, the centre



of the circle, draw *OL* at right angles to the diameter *MN*, and cutting the circumference in *L*. Join *BO*, *AO*.

The area of the triangle *AOB* is (I. 41) half that of the rectangle *ABDC*; therefore its area is known. But its sides *OA, OB* are constant, since *OA, OB* are radii of the circle: hence the problem is reduced to construct the triangle *AOB*, so that its base, *AB*, shall be parallel to the diameter *MN*, and its area shall be equal to that of half the given rectangle.

To do this: construct (see Ex. 1, Prop. 48) the triangle *EFG*, having its sides, *EF, FG*, each equal to the radius of the circle, and its area equal to half the given area of the rectangle.

Now, at the centre, *O*, draw *OB*, making the angle *BOL* equal to half the angle *EFG*; and draw *OA*, making the angles *AOL, BOL* equal. Join *A, B*, where *OA, OB* meet the circumference. Draw *AC, BD* perpendicular to the diameter *MN*. *ABCD* is the required rectangle.

For, clearly (I. 4), the triangles *AOB, EFG* are equal in every respect; therefore *AOB* is half the given area; and since *OL*, by construction, bisects the angle *BOA, AB* is (I. 4) at right angles to *OL*: wherefore *ABCD* is a rectangle, and it is also equal to double the triangle *AOB*, or the given area.

To solve (2).

Let the construction be the same. If *ABCD* is a maximum, clearly the triangle *AOB* is a maximum. But the triangle *AOB* is a maximum, only, when the sides *OA, OB* are at right angles (73). Hence the construction is simply to draw *OA, OB* making the angles *AOL, BOL*. each half a right angle, and joining *A, B*, the points where *OA, OB* meet the circumference; and finally drawing

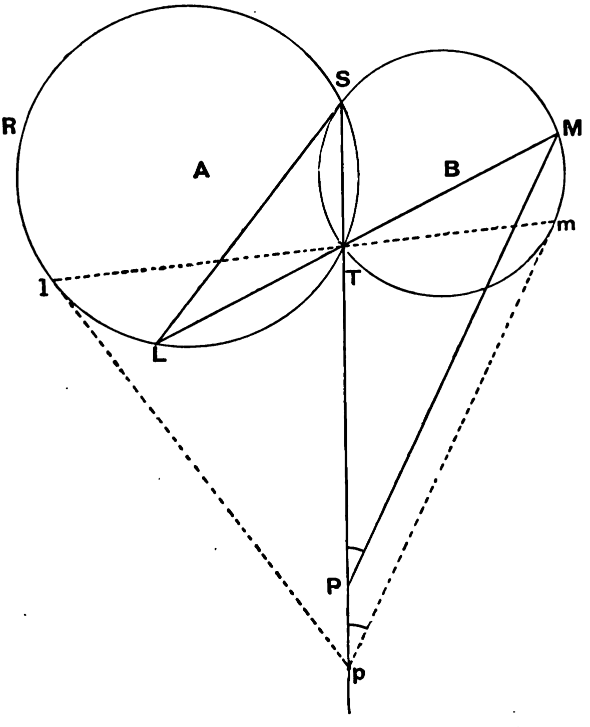
AC, *BD* perpendicular to *MN*. *ABCD* is, then, clearly, a rectangle, and of maximum area.

Prop. 80. Through either point of intersection of two given circles, it is required to draw a straight line cutting them, so that of the intercepted chords:—

(1) The rectangle under them shall be given.

(2) The rectangle under them shall be a maximum.

Let (*A*), (*B*), be two circles intersecting in *S*, *T*: it is required (1) to draw through either point, *T*, a straight



line LM, so that $LT \, . \, TM$ shall be equal to a given square: (2) so that $lT \, . \, Tm$ shall be the greatest possible.

To solve (1).

Upon ST, produced, take TP, so that $ST \, . \, TP$ is equal to the given square. At the point, P, with the straight line PT, make the angle TPM equal to the constant angle in the segment SRT. Where PM cuts the circle (B) in M, join MT, and let MT, produced, cut the circle (A) in L. Join LS.

Since, by construction, the angles SLT, TPM, are equal, a circle will pass through the points S, L, P, M (15). Hence (III. 35), $ST \, . \, TP = LT \, . \, TM$, and, therefore, $LT \, . \, TM$ is equal to the given square.

To solve (2).

From the first part, we see that $LT \, . \, TM = ST \, . \, TP$, where ST is fixed. Hence, if $LT \, . \, TM$ is to be the greatest possible, $ST \, . \, TP$ must be the greatest possible; that is, TP is to be the greatest possible.

Now, the angle TPM is constant; therefore TP will be the greatest possible when MP, moving parallel to itself, becomes the tangent mp. Join mT, and produce it to meet the circle (A) in l. $lT \, . \, Tm$ is, then, clearly equal to $ST \, . \, Tp$, and therefore a maximum.

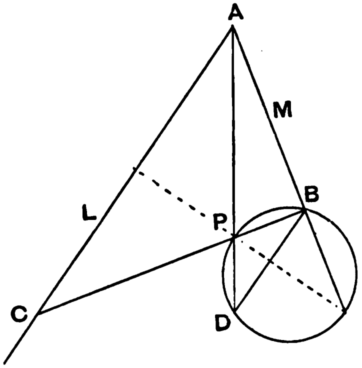
Note. It will be found, without any difficulty, that, since mp is a tangent to the circle (B), lp is also a tangent to the circle (A): remembering that the angles Tpm, SlT are equal.

PROP. 81. Through a point, given in position, either within or without a fixed angle, to draw a straight line cutting the legs, so that, of the intercepted segments:—

(1) The rectangle under them shall be given.

(2) The rectangle under them shall be a minimum.

Let *LAM* be the fixed angle, *P* the fixed point: it is required to draw through, *P*, a straight line *BPC*, so that *PB* . *PC* (1) shall be equal to a given square; (2) shall be equal to the least square possible.



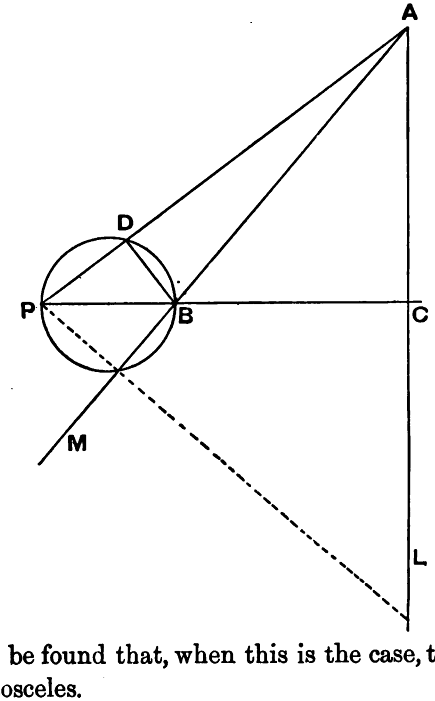
(1) Join *PA*; and upon *PA*, produced, if necessary, take *PD*, so that, *PA* . *PD* shall be equal to the given square.

Upon *PD* describe the segment of the circle *PDB*, containing an angle equal to the fixed angle *PAC*.

Let this segment cut the leg *AM* in *B*. Join *BP*, and produce it to meet *AL* in *C*:

PB . *PC* is the required rectangle. For, the angles *PBD*, *PAC* being equal, by construction, therefore (15, 16), (III. 35), *PB* . *PC* = *PA* . *PD*, that is, the given square.

To solve (2).* Clearly *PB* . *PC* will be a minimum when *PA* . *PD* is a minimum, or when *PD* is a minimum. Now the angle *PBD* is constant, hence *PD* will be a minimum when the segment *PBD* is a minimum, which is clearly the case when the circle touches the line *AL*.



It will be found that, when this is the case, the triangle *ABC* is isosceles.

For, the angles *PBM*, *PDB* are, then, equal (III. 32), and the angles *PDB*, *ACB* are also equal; therefore the angles *PBM*, *ACB* are equal, or the triangle *ABC* is isosceles.

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* For another solution of this, see the "Educational Times" for December, 1881.

EXAMPLES.

1. Find the point in the circumference of a circle the sum of its distances from two fixed points in the same circumference shall be a maximum.

[The solution here is identically the same as that in Prop. 72, with the exception that the angle subtended at the circumference of the circle by the two fixed points is not necessarily a right angle : the point required is, then, the bisection of the arc.]

2. Of all triangles that can be inscribed in a circle the equilateral has the maximum perimeter.

[Considering the foregoing, the reasoning is similar to that used in Prop. 87.]

3. Prove, generally, that of all figures inscribable in a circle the regular has the maximum perimeter.

[The same reasoning here, again, applies.]

4. Given, of a triangle, the base and vertical angle: construct it, so that its area shall be a maximum.

[The area is clearly a maximum when the vertex of the triangle is the bisection of the arc of the segment of the circle described upon the given base, and containing an angle equal to the given vertical angle.]

5. Of all triangles that can be inscribed in a given circle, the equilateral has the maximum area.

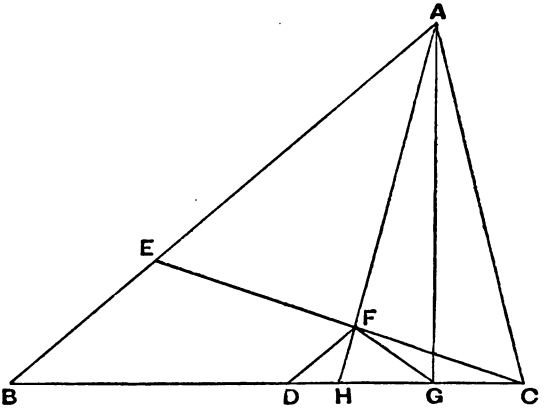
[This follows from the foregoing, by a reasoning similar to that in Prop. 78.]

6. Of all figures that can be inscribed in a given circle, the regular has the maximum area.

[The same reasoning here, also, applies.]

MISCELLANEOUS PROPOSITIONS.

Prop. 82. The rectangle under the distances of the middle point of the base of a triangle from the foot of the perpendicular, and the point where the internal bisector of the vertical angle meets the base, is equal to the square of half the difference of the sides.



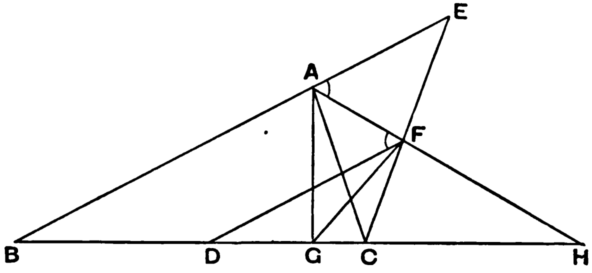
Let ABC be any triangle, of which AG, AH are, respectively, the perpendicular, and internal bisector of the vertical angle, meeting the base in G, H. Bisect BC in

D: then *DG* . *DH* is equal to the square of half the difference of the sides.

Cut off *AE* equal to *AC*, and join *CE*, meeting *AH* in *F*. Join *DF*.

Since the angles *CAF*, *EAF* are equal by hypothesis, and the sides *CA*, *AE* also equal; therefore (I. 4) *CF*, *EF* are equal, and *AF* is perpendicular to *CE*. Hence (1) *DF* is parallel to *BE*, and equal to half of it, that is, *DF* is half the difference of the sides. Again, the angles *DFH*, *BAH* are equal (I. 29); and the angles *BAH*, *CAF* are equal (hyp.); but the angles *CAF*, *HGF* are equal (17), since a circle will circumscribe the quadrilateral *AFGC* (the angles *AFC*, *AGC* being right); therefore the angles *DFH*, *HGF* are equal. Hence (31) $DG \,.\, DH = DF^2$, that is, the square of half the difference of the sides.

Prop. 83. The rectangle under the distances of the middle point of the base of a triangle from the foot of the perpendicular, and the point where the external bisector of the vertical angle meets the base produced, is equal to the square of half the sum of the sides.



Let *ABC* be any triangle, of which *AG*, *AH* are respectively, the perpendicular, and external bisector of the vertical angle, meeting the base in *G*, *H* : *DG* . *DH* is equal to the square of half the sum of the sides, *D* being the middle point of the base *BC*.

Produce *BA*, and take off *AE* equal to *AC*. Join *CE*, meeting *AH* in *F*. Join *FG*, *DF*.

Since *AE*, *AC* are equal, by construction, and the angles *CAF*, *EAF* are also equal; therefore (I. 4) *CF*, *EF* are equal, and *AF* is at right angles to *CE*. Hence (1) *DF* is parallel to *BE*, and equal to half of it, that is, half the sum of the sides, for *BE* is equal to the sum of *BA*, *AC*.

Again, the angle *AHC* is (I. 32) equal to the difference between the angles *ACG*, *CAH*, that is, the difference between the angles *AFG*, *CAH* (since the quadrilateral *AGCF* is clearly circumscribable by a circle, the angles *ACG*, *AFG* are equal (III. 21)), or, the difference between *AFG*, *EAF*, since *CAH*, *EAF* are equal, that is, between *AFG*, *AFD* (since *BE*, *DF* are parallel), which is the angle *GFD*. Hence (31) $DG . DH = DF^2$, or the square on half the sum of the sides.

EXAMPLES.

1. Given, of a triangle, the internal bisector of the vertical angle, the perpendicular from the vertex on the base, and the difference of the sides : construct it.

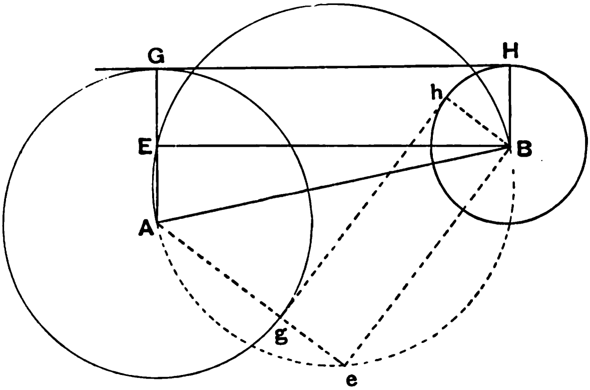
[Let (see fig. Prop. 82) *AH*, *AG*, be, respectively, the given internal bisector of the vertical angle, and the perpendicular. With these lines construct the right-angled triangle *AHG*. Now, since the difference of the sides is given, clearly the rectangle *DG* . *DH* is known; and the difference of the lines *DG*, *DH*, being the fixed line *HG*, therefore the lines *DG*, *DH* are (33) determined. Hence the middle point of the base, *D*, is determined. Inflect, then, *DF*, half the given difference of the sides, over to

AH, meeting it in *F*. The remainder of the solution and after proof is clear from Prop. 82.]

2. Given, of a triangle, the external bisector of the vertical angle, the perpendicular from the vertex to the base, and the sum of the sides: construct it.

[Consider Prop. 83.]

PROP. 84. To draw a common tangent to two given circles.



Let *A*, *B* be the centres of the given circles: suppose *GH* a common tangent to both circles, *G*, *H* being the points of contact. Join *AG*, *BH*, and draw *BE* parallel to *HG*, meeting *AG* in *E*; then *BEGH* is obviously a rectangle; hence *EG* is equal to the radius of the circle (*B*), and, therefore, *AE* is equal to the difference between the radii of the two circles. And the angle *AEB* is clearly a right angle. Hence, then, the following construction:—

Upon *AB* describe a semicircle. From *A* inflect over *AE* to the circumference, and equal to the difference of the two radii. Let *AE*, produced, meet the circumference of the circle (A) in G. Through B draw BH parallel to AG, and meeting the circumference of (B) in H. Join GH, BE.

Since AE is, by construction, equal to the difference between AG, BH, and it is also the difference between AG, EG, the lines EG, BH are equal; but they are also parallel; therefore the figure $BEGH$ is a parallelogram, and therefore the angles AEB, AGH are (I. 29) equal. But the angle AEB is right, being in a semicircle; therefore the angle EGH is right, whence also the angle BHG is right, in other words, GH is a common tangent to the two circles.

Note. If, instead of inflecting AE in the semicircle equal to the difference of the radii, we inflected Ae equal to their sum, the points g, h, where Ae meets the circle (A), and Bh, drawn parallel to Ae, meets the circle (B), will be the contacts of another common tangent gh. The common tangents, GH, gh, are called, respectively, *direct*, and *transverse*. Evidently there are, in general, two of each kind.

EXAMPLES.

1. By means of Propositions 3, and 59, draw a common tangent to two given circles.

2. If d denote the distance of the line joining the centres of two circles of radii r', r'', respectively: prove that t, their common tangent, is given from the equation

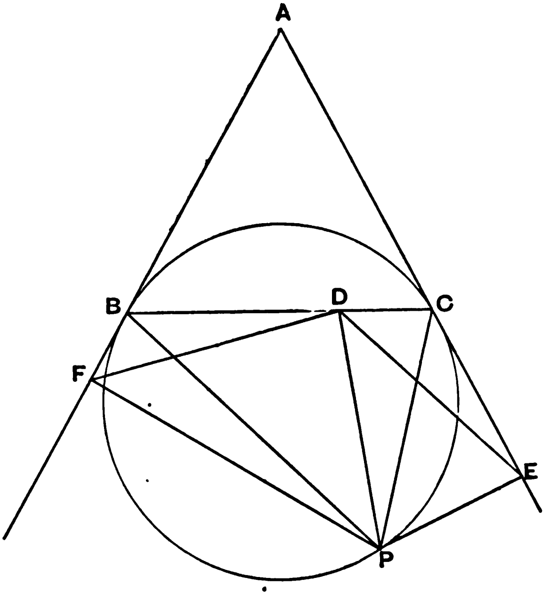
$$t = \sqrt{d^2 - (r' - r'')^2}.$$

3. If two circles touch, prove that the square of their

common tangent is equal to the rectangle under the diameters of the circles.

[Though this follows from the preceding example, immediately, it is far better proved purely geometrically.]

PROP. 85. If, from any point on the circumference of a circle, perpendiculars be drawn to two tangents, and to their chord of contact, the rectangle under the first two perpendiculars is equal to the square of the latter.



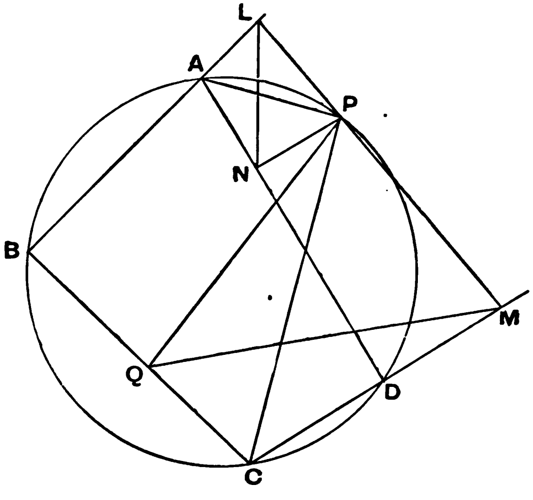
Let *AB*, *AC*, be any two tangents to a given circle; *P* any point on its circumference; *PD*, *PE*, *PF*, perpendiculars to *BC*, the chord of contact, *AC*, *AB*, respectively: then $PE \cdot PF = PD^2$.

Join *PC, PB, FD, DE.*

Since the angles *PDC, PEC* are each right, the quadrilateral *PECD* is circumscribable by a circle; therefore (III. 21) the angles *PDE, PCE* are equal. But the angles *PCE, PBC* are equal (III. 32); therefore the angles *PDE, PBD* are equal. And, since the quadrilateral *PFBD* is circumscribable by a circle, the angles *PBD, PFD* are equal. Hence the angles *PDE, PFD* are equal. In a similar manner, the angles *PED, PDF* can be proved equal; therefore the triangles *PDE, PFD* are equiangular; and therefore (VI. 4),

$$PE : PD :: PD : PF; \text{ or } PE \,.\, PF = PD^2 \text{ (VI. 16).}$$

PROP. 86. If, from any point on the circumference of a circle, perpendiculars be drawn to the sides of an inscribed quadrilateral, the rectangle under the perpendiculars to two opposite sides is equal to the rectangle under the perpendiculars to the other sides.



Let P be any point on the circumference of the circle circumscribing the quadrilateral $ABCD$; and let PL, PM, PN, PQ, be perpendiculars to the sides of $ABCD$. Join AP, PC, LN, and QM.

The quadrilaterals $LANP$, $CMPQ$ are (16) circumscribable by circles, since, of each, their opposite angles are together equal to two right angles.

Hence (III. 21) and (17) the angle LNP is equal to the angle LAP, is equal to the angle PCB (or PCQ), is equal to the angle PMQ. Similarly it can be shown that the angles PLN, PQM are equal. Therefore the triangles PLN, PQM are equiangular, whence (VI. 4)

$$PL : PN :: PQ : PM, \text{ or}$$

$$PL \,.\, PM = PN \,.\, PQ \quad \text{(VI. 16)}.$$

EXAMPLES.

1. Deduce Proposition 85 from Proposition 86.

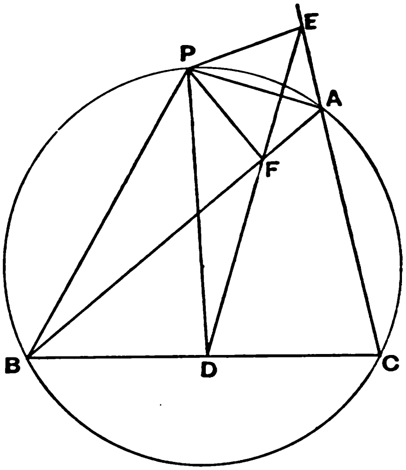
2. Find the point P, in Proposition 85, where $PE \,.\, PF$ is a maximum. Show that there are two such points.

3. Find the point P, in Proposition 85, where $PE \,.\, PF$ is equal to a given square.

PROP. 87. If, from any point on the circumference of a circle, perpendiculars be drawn to the sides of an inscribed triangle, their feet shall lie on the same straight line.

Let PD, PE, PF, be perpendiculars from any point P on the circumference of a circle to the sides, BC, CA, AB, respectively, of the inscribed triangle ABC. Join DF, FE : EFD is one straight line. Join PA, PB.

The quadrilaterals *PFAE*, *PFDB* are (16, 15) clearly inscribable by circles. Therefore (III. 21) the angles



AFE, *APE* are equal. Similarly the angles *BFD*, *BPD* are equal. But, in the triangles *BPD*, *PAE*, the angles *PDB*, *PEA* are equal, each being right; and the angles *PAE*, *PBC* are equal (17); therefore the remaining angles *BPD*, *APE* are equal; and therefore their equivalents *BFD*, *AFE* are equal. But *AB* is a straight line, therefore (I. 15), *DFE* is a straight line.

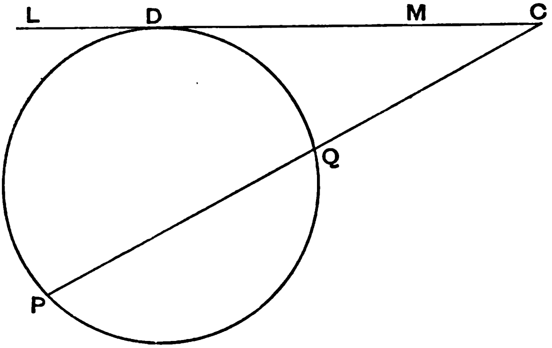
PROP. 88. To describe a circle passing through two given points and touching

(1) A given straight line.

(2) A given circle.

(1) Let *P*, *Q* be the given points, *LM* the giv

straight line. Join PQ, and let the straight lines PQ, LM intersect in C. From CL cut off CD equal to a side



of the square that the rectangle $CP.CQ$ is equal to (II. 14). Describe a circle about the triangle PDQ: this circle touches the straight line LM in D.

For, $CP.CQ = CD^2$; and, therefore (III. 37), CD is a tangent to the circle.

(2) Let P, Q, as before, be the given points, and (A) the given circle.

Describe any circle, $PQEF$, passing through the given points, and cutting the circle (A) in E, F. Join E, F, and P, Q; and let PQ, EF, produced, meet in C. From C draw CD tangent to the circle (A): the circle described about the triangle DPQ is the required one.

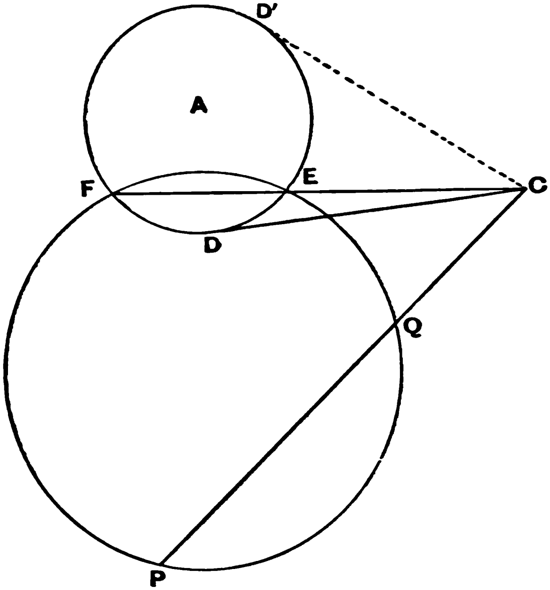
From the circle $PFEQ$, we have (III. 36)

$$CE.CF = CP.CQ;$$

and from the circle (A), we have also (III. 36)

$$CE.CF = CD^2;$$

therefore $CP.CQ = CD^2$; and therefore (III. 37) CD is a



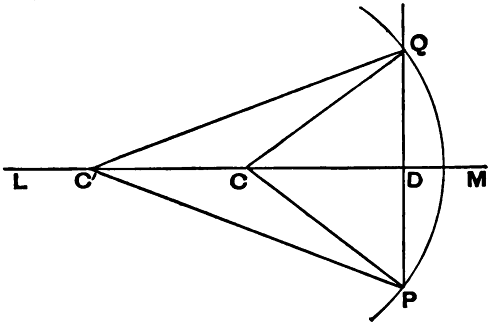
tangent to the circle described about the triangle PDQ. But it is also a tangent to the circle (A); therefore the two circles touch at this point D.

Note. Since, in general, two tangents can be drawn to a given circle from a given point, it follows that two circles can, in general, be drawn passing through two given points and touching a given circle.

Prop. 89. Every circle, passing through a given point, and having its centre on a given straight line, passes also through another fixed point.

Let P be the given point, LM the given straight line.

Draw *PD* perpendicular to *LM*, and produce it, cutting off *DQ* equal to *PD*; then, *Q* is a fixed point.



Take any point *C* on *LM*. Join *CP*, *CQ*. In the triangles *CDP*, *CDQ*, the sides *CD*, *DP* are equal to the sides *CD*, *DQ*, respectively, and the angles *CDQ*, *CDP* are equal, each being right; therefore (I. 4) the sides *CP*, *CQ* are equal. Hence the circle with centre *C*, and radius *CP*, passes through the fixed point *Q*.

EXAMPLES.

1. Describe a circle passing through a given point, having its centre on a given straight line, and touching another given straight line.

[From Prop. 89, the problem is seen to be immediately reduced to Prop. 88, (1).]

2. Describe a circle passing through a given point, having its centre on a given straight line, and touching a given circle.

3. Describe a circle passing through a given point and touching two straight lines.

[Since the centre of every circle touching two straight lines lies on a bisector of the angle between them, the problem is plainly reduced, finally, to Prop. 88, (1).]

4. Prove that in figures (1), (2), Prop. 88, the points P, Q, subtend a maximum angle at the point D*.

5. Prove that in Prop. 88, (2), the points P, Q, subtend a minimum angle at the point D', D' being the point of internal contact of the circles.

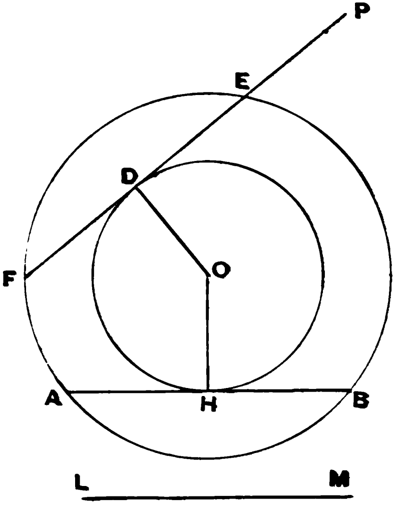
[The solution of this and the preceding Example follows immediately from (I. 16) and (III. 21.)]

6. Prove that the point C, in Prop. 88, (2), is a fixed point.

7. In the particular case (Prop. 88) where the point Q is on the straight line LM, or on the circumference of the circle (A), solve the problems.

8. In the particular case where the line joining P, Q is parallel to the line LM (Prop. 88, (1)), solve the problem.

Prop. 90. From a given point it is required to draw a straight line cutting a given circle, so that the intercepted chord shall be of given length.



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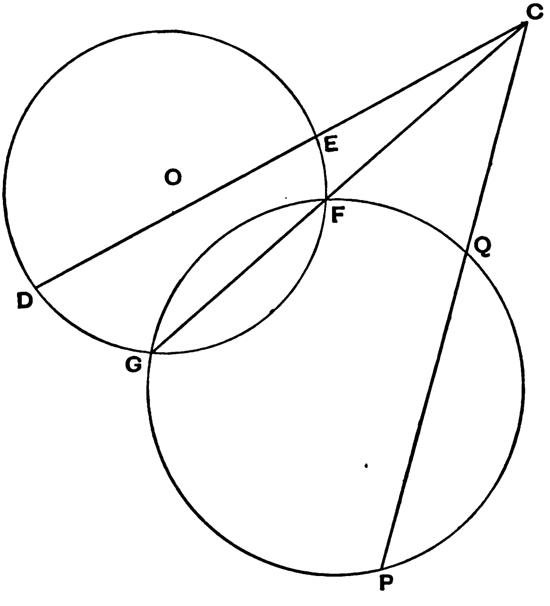
* The point D, in the case of the circles, is supposed to be the point of external contact.

Let *P* be the given point, (*O*) the given circle, and *LM* the given length.

Take any chord *AB*, in the circle (*O*), equal to *LM*. Draw *OH* perpendicular to *AB*, and with this distance as radius, and *O* as centre, describe a circle.

From *P* draw *PD* a tangent to this circle, and cutting the outer circle in *E*, *F*: *PEF* is the required line. Join *OD*. The chords *AB*, *EF* are clearly equally distant from the centre *O*, and therefore (III. 14) equal. Therefore *EF* is equal to the given length *LM*.

Prop. 91. To describe a circle passing through two given points, and intercepting a given arc on a given circle.

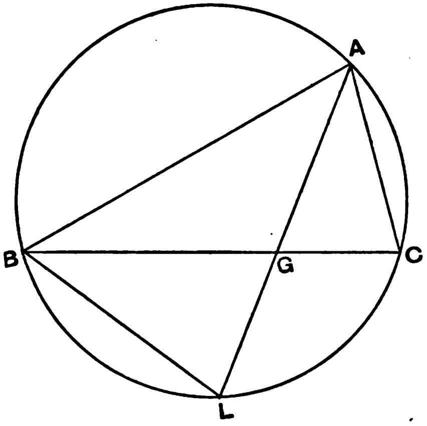


Let *P*, *Q* be the given points; (*O*) the given circle: it is required to describe a circle passing through P, Q, and intercepting a given arc on the circle (O).

Describe any circle passing through P, Q, and cutting the given circle (O) in G, F. Let PQ, GF, produced, meet in C. Through C draw, by the preceding, a straight line CED cutting the circle (O), and so that the intercepted chord ED shall be equal to the chord that the given arc subtends.

Since (III. 36) $CE \cdot CD = CF \cdot CG$ from the circle (O), and $CF \cdot CG = CP \cdot CQ$ from the circle $PQFG$; therefore $CE \cdot CD = CP \cdot CQ$. Hence (17) a circle will pass through $PQDE$. Evidently this is the required circle, since the arc $DGFE$ is equal to the given arc (III. 28).

Prop. 92. The rectangle contained by the sides of a triangle is equal to the rectangle contained by the segments of the base, made by the internal bisector of the vertical angle, together with the square on that bisector.



Let ABC be any triangle, circumscribed by a circle, of which AG is the internal bisector of the angle A, and which meets the base BC in G, and the circumference in L: $BA \,.\, CA = BG \,.\, CG + GA^2$. Join BL.

Since the angles BAL, CAG are equal by hypothesis, and the angles BLA, GCA, being in the same segment, the triangles BAL, CAG are equiangular. Therefore (VI. 4)

$$BA : AL :: AG : AC;$$

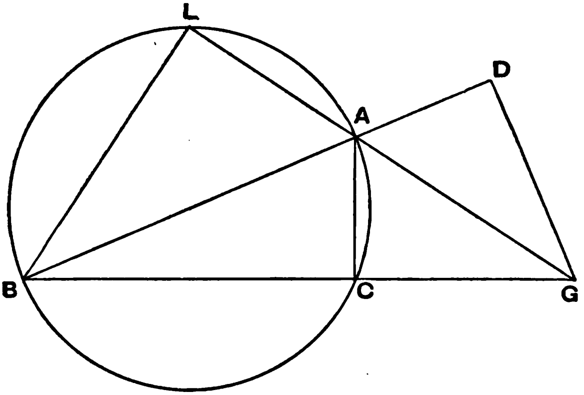
∴ (VI. 16) $$BA \,.\, CA = AL \,.\, AG.$$

But (II. 3) $$AL \,.\, AG = AG^2 + AG \,.\, GL,$$

or, $$= AG^2 + BG \,.\, GC \quad \text{(III. 35)};$$

$$\therefore BA \,.\, CA = AG^2 + BG \,.\, GC.$$

PROP. 93. The rectangle contained by the sides of a triangle is equal to the rectangle contained by the segments of the base, made by the external bisector of the vertical angle, less by the square of that bisector.

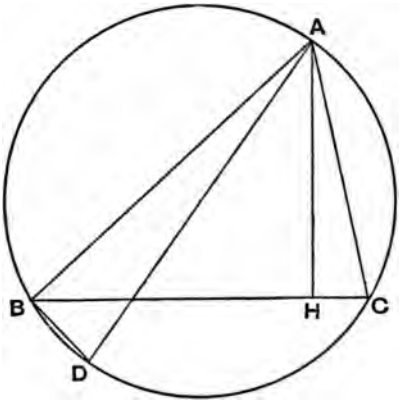


Let ABC be any triangle, circumscribed by a circle, of which AG is the external bisector of the angle A, and which meets the base BC, produced, in G, and the circumference in L: $BA \,.\, AC = BG \,.\, CG - AG^2$. Produce BA, and take AD equal to AC. Join GD, BL.

The triangles CAG, ADG are, clearly (I. 26), equal in every respect; therefore the angles ADG, ACG are equal. But the angles ACG, ALB are equal (17); therefore the angles ADG, ALB are equal, whence (15) a circle will circumscribe the quadrilateral $BLDG$. Therefore (III. 35) $BA \,.\, AD = GA \,.\, AL$; to each of these equals add AG^2,

and $BA \,.\, AD + AG^2 = GA \,.\, AL + AG^2$;
but $GA \,.\, AL + AG^2 = GL \,.\, GA$ (II. 3),
that is, $GB \,.\, GC$ (III. 36).
Hence, $BA \,.\, AD + AG^2 = GB \,.\, GC$,
or, $BA \,.\, AC + AG^2 = GB \,.\, GC$.

PROP. 94. The rectangle contained by the sides of a



triangle is equal to the rectangle contained by the diameter of the circumscribing circle and the perpendicular from the vertex to the base.

Let ABC be a triangle, circumscribed by a circle, and let AH be the perpendicular from the vertex A to the base BC. Draw AD the diameter: then

$$AD\,.\,AH = AB\,.\,AC.$$

Join BD. Since the angle in a semicircle is right, the angles ABD, AHC are equal. And the angles ADB, ACH are equal (III. 21); therefore the triangles ABD, ACH are equiangular; whence (VI. 4)

$$AB : AD :: AH : AC;$$

or, $$AB\,.\,AC = AD\,.\,AH \quad \text{(VI. 16)}.$$

EXAMPLE.

Given, of a triangle, the base, vertical angle, and the rectangle contained by the sides: construct it.

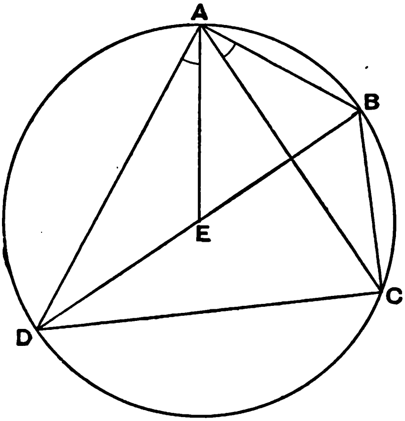
PROP. 95. The rectangle contained by the diagonals of a quadrilateral inscribed in a circle is equal to the sum of the rectangles contained by the opposite sides.

Let $ABCD$ be a quadrilateral inscribed in a circle: then $$AC\,.\,BD = AB\,.\,CD + BC\,.\,AD,$$ where AC, BD are the diagonals.

Draw the straight line AE, meeting BD in E, and making the angle DAE equal to the angle BAC.

Since the angles ADE, BCA are equal (III. 21), and the angles DAE, BAC, by construction, the triangles DAE, BAC are equiangular. Therefore (VI. 4) $AD : DE :: AC : BC$; or (VI. 16), $AD\,.\,BC = DE\,.\,AC$.

Again, the triangles BAE, ADC are clearly equiangular;



therefore (VI. 4) $AB : BE :: AC : CD$;

or (VI. 16), $AB . CD = AC . BE$.

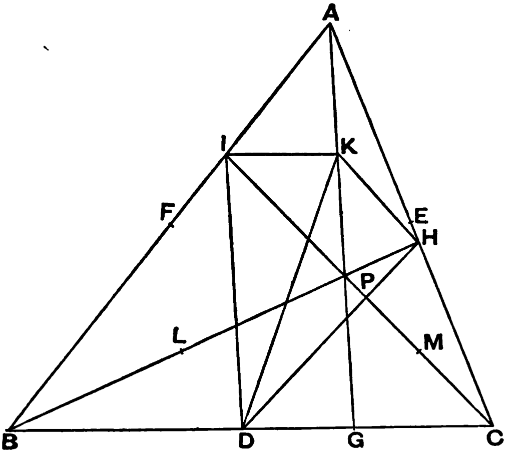
Hence, by addition, $AB . CD + AD . BC$

$= DE . AC + BE . AC = (DE + BE) AC = BD . AC$.

Prop. 96. The circle which passes through the feet of the perpendiculars of a triangle drawn from the vertices to the opposite sides passes also through the middle points of the sides, and through the middle points of the upper segments of these perpendiculars.

Let ABC be any triangle. Draw AG, BH, CI perpendiculars to the opposite sides, meeting in the point P. Bisect AP in K, and BC in D. Join DI, KI, DH, KH.

Since K is the middle point of AP, and the angle AIP is a right angle, the lines KI, KP, are equal (2); therefore (I. 5) the angles KIP, KPI are equal, that is, the angles KIP, GPC are equal (I. 15). And since the angle BIC is right, and D is the middle point of BC, the angles *DIC*, *DCI* are equal. Therefore the sum of the angles *KIP*, *DIC*, which is *DIK*, is equal to the sum of



the angles *GPC*, *DCI*, which is clearly a right angle (I. 32). Hence, then, the angle *DIK* is a right angle. In a similar manner, the angle *DHK* can be shown to be a right angle: and the angle *KGD* is a right angle by hypothesis; therefore (16) the same circle will circumscribe the quadrilaterals *DGKI*, *DHKI*. Thus, then, the circle passing through the feet of the perpendiculars passes also through the middle point of any one side, and through the middle point of the upper segment of the corresponding perpendicular. That is to say, the same circle passes through the feet of the perpendiculars, the middle points of the sides, and the middle points of the upper segments of these perpendiculars.

Note. Since this circle passes through these nine particular points, it is called the nine-point circle.

PROP. 97. Given the base of a triangle, and the vertical angle: to find the locus of the centre of the nine-point circle.

Let the construction be the same as in Prop. 96.

Since the nine-point circle passes through the middle points of the sides, its radius is half that of the circumscribing circle of the triangle *ABC**. But since the base, and vertical angle, of the triangle *ABC* are given, the radius of the circumscribing circle is (III. 33) given. Hence *DK*, which is obviously the diameter of the nine-point circle, is equal to the radius of the circle about *ABC*, and therefore given. The locus, then, of the middle point

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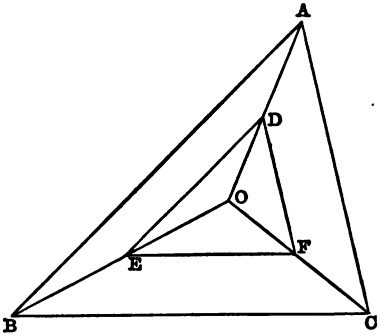
* This follows from the fact that if any point, *O*, be joined to the vertices of a triangle *ABC*, and if *D*, *E*, *F* be the middle points of *OA*, *OB*, *OC*, respectively, then (1) the triangle *DEF* is the same as that formed by joining the middle points of the sides of the triangle. If, now, *O* be the centre of the circumscribing circle of the triangle *ABC*, it is also the centre of the circumscribing circle of the triangle *DEF*, since *OD*, *OE*, *OF* are each half, respectively, of the equals *OA*, *OB*, *OC*. The truth of the above statement is, therefore, manifest.



The theorem is, of course, general, for *O*, being taken as any special point for the triangle *ABC*, is, also, the corresponding point for the triangle *DEF*.

of DK, is a circle, with centre D, and radius equal to half the given radius of the circle about the triangle ABC.

EXAMPLES.

1. Prove that the centre of the nine-point circle is the point of bisection of the straight line joining the centre of the circumscribing circle with the intersection of the perpendiculars.

[Consider Euclid (III. 3).]

2. Show that the triangles (fig. Prop. 96) BPC, CPA, APB have the same nine-point circle.

3. Given, in position, the centre of the circumscribing circle of a triangle, the centre of its nine-point circle, and the middle point of the base : construct it.

[Consider Ex. 1, and Prop. 27.]

4. Given, in position, the centre of the nine-point circle of a triangle, the intersection of the perpendiculars, and the middle point of the base: construct the triangle.

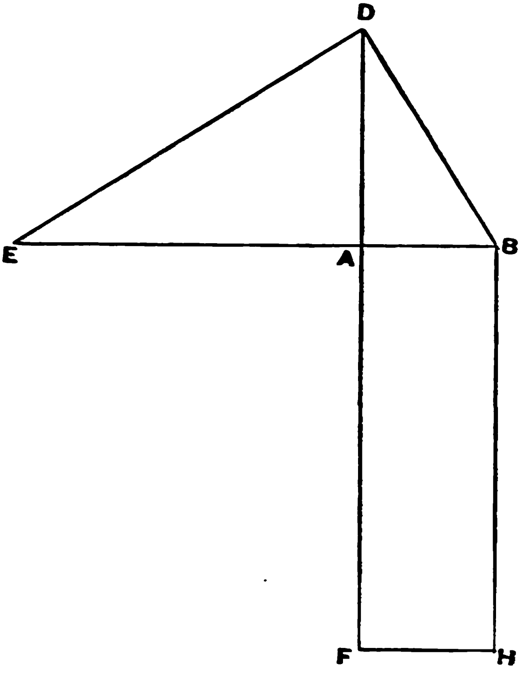
PROP. 98. Given the rectangle under two lines and the difference of their squares: to find the lines.

Let AB be a line, the square on which is equal to the difference of the squares of the lines we are seeking; and let the rectangle $ABHF$, described upon AB, be equal to the rectangle under the lines. Produce BA to E, so that $BE \,.\, EA$ shall be equal to the square on AF (33); from E inflect ED, equal to FA, over to FA produced, meeting it in D; join BD: AD, BD are the required lines.

The rectangle $BE \,.\, EA = AF^2$, by construction; but AF, ED are also equal, by construction; therefore

$$BE \,.\, EA = ED^2,$$

and, therefore (III. 37), ED is a tangent to the circumscribing circle of the triangle DBA; but BD is the diameter of this circle, since the angle BAD is right; therefore (III. 16) EDB is a right angle. Hence (6) the triangles EAD, BAD are equiangular; therefore (VI. 4) $AD : ED :: AB : BD$, or (VI. 16) $AD . BD = AB . ED$, or $AD . BD = AB . AF$, that is, the given rectangle.



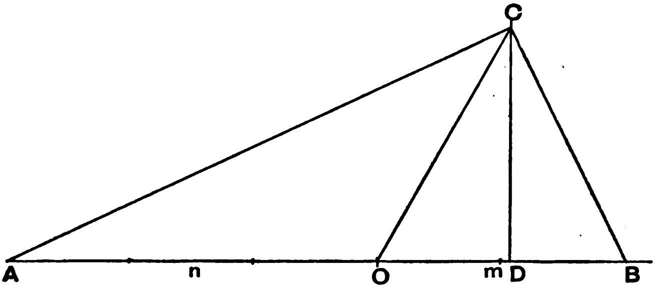
Again, since the angle BAD is right,

$$BD^2 - AD^2 = AB^2 = \text{the given square.}$$

(The above solution I have taken from Lardiner's Euclid.)

Prop. 99. ABC is a triangle, of which the base AB is divided into $m+n$ equal parts; if AO contain n equal parts, and BO m parts, to prove that, if the vertex C be joined to O,

$$mAC^2 + nBC^2 = mAO^2 + nBO^2 + (m+n)\ OC^2.$$



Draw CD perpendicular to the base. Then (II. 12)

$$AC^2 = AO^2 + CO^2 + 2AO\,.\,OD;$$

and (II. 13) $BC^2 = BO^2 + CO^2 - 2BO\,.\,OD.$

Multiplying the first equation by m, and the second equation by n, and adding, we have

$$mAC^2 + nBC^2 = mAO^2 + nBO^2 + (m+n)\,CO^2 \\ + 2OD\,(mAO - nBO).$$

But mAO obviously equals nBO, since BO contains m, and AO n equal parts; therefore the last term of the equation, viz. $2OD\,(mAO - nBO)$, vanishes. Hence

$$mAC^2 + nBC^2 = mAO^2 + nBO^2 + (m+n)\,OC^2.$$

Example.

Given, in magnitude and position, the base of a triangle, and that m times the square of one side together with n times the

square of the other is equal to a given square; find the locus of the vertex.

[Divide the base, AB, into $m+n$ equal parts (18); let AO contain n parts, BO m parts; then AO, BO, and the point O are fixed. Hence, in the above equation, the only unknown quantity is OC, which can immediately be determined. The locus is, then, a circle, with O as centre, and a radius OC of known length.]

Prop. 100. The area of a triangle is equal to

$$\sqrt{s(s-a)(s-b)(s-c)},$$

where s is equal to half the perimeter.

Let (see fig. Prop. 23) O be the centre of the inscribed circle, and O_1 the centre of the escribed circle opposite the angle A, of the triangle ABC. Draw Of, O_1F perpendiculars to AB.

Since (5) the angle OBO_1 is right, the sum of the angles OBf, O_1BF is equal to a right angle; but the sum of the angles OBf, BOf is also a right angle; therefore the angles O_1BF, BOf are equal; and the angles BfO, BFO_1 are equal, each being right; therefore the triangles BOf, BO_1F are equiangular. Hence (VI. 4)

$$Of : Bf :: BF : O_1F,$$

or $\quad r : s-b :: s-c : r_1$ (see Prop. 23, Exs. 1, 2),

that is, $\quad rr_1 = (s-b)(s-c).$

But $sr = (s-a)r_1 = \Delta$, the area of the triangle (see Prop. 23, Exs. 4, 5); therefore

$$s.(s-a)rr_1 = \Delta^2; \text{ but } rr_1 = (s-b)(s-c),$$

therefore $\quad s.(s-a)(s-b)(s-c) = \Delta^2,$

or $\quad \Delta = \sqrt{s.(s-a)(s-b)(s-c)}.$

EXAMPLES.

1. If Δ denote the area of a triangle, and r, r_1, r_2, r_3 the radii of its inscribed and escribed circles respectively, prove that $\Delta = \sqrt{rr_1r_2r_3}$.

[Since (see Exs. 4, 5, page 27)

$$\Delta = sr = (s-a)\,r_1 = (s-b)\,r_2 = (s-c)\,r_3,$$

$$\therefore \Delta^4 = s\,(s-a)\,(s-b)\,(s-c)\,rr_1r_2r_3;$$

but (100)

$$\Delta^2 = s\,(s-a)\,(s-b)\,(s-c),$$

$$\therefore \Delta^2 = rr_1r_2r_3, \text{ or } \Delta = \sqrt{rr_1r_2r_3}\,].$$

2. Prove that $$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}.$$

[Since $sr = \Delta$, therefore $$\frac{1}{r} = \frac{s}{\Delta}.$$

Similarly $\frac{1}{r_1} = \frac{s-a}{\Delta}$, $\frac{1}{r_2} = \frac{s-b}{\Delta}$, and $\frac{1}{r_3} = \frac{s-c}{\Delta}$;

therefore, by addition,

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{s-a+s-b+s-c}{\Delta} = \frac{3s-(a+b+c)}{\Delta} = \frac{3s-2s}{\Delta} = \frac{s}{\Delta} = \frac{1}{r}.]$$

3. If the radii of any two escribed circles are equal, prove that the triangle must be isosceles.

4. Prove that, of an equilateral triangle, the radius of an escribed circle is equal to three times the radius of its inscribed.

5. Prove, geometrically, that if the radius of an escribed circle of a triangle is three times the radius of the inscribed, the sides of the triangle are in Arithmetical progression.

[Examples 3, 4, 5 follow clearly from the Algebraical formulæ, viz: $sr=(s-a)r_1=(s-b)r_2=(s-c)r_3$; however, to arrive at the solutions by this method would be quite absurd.]

Examples on (I. 4).

1. In an isosceles triangle, the straight line which bisects the vertical angle bisects also the base, and is perpendicular to it.

2. The straight line joining the vertices of two isosceles triangles standing on the same base bisects this common base, and is at right angles to it.

3. If the diagonals of a quadrilateral bisect each other, it is a parallelogram.

4. If at the middle point of a finite straight line a perpendicular be erected, prove that the distances of the two ends of the finite line from any point on the perpendicular are equal.

5. ABC is an equilateral triangle, and points P, Q, R are taken on the sides BC, CA, AB, respectively, such that CP, AQ, BR are each equal; prove that the triangle PQR is equilateral.

6. $ABCD$ is a square and points P, Q, R, S are taken on the sides AB, BC, CD, DA, respectively, such that AP, BQ, CR, DS are each equal; prove that $PQRS$ is a square.

Examples on (I. 8).

1. If the opposite sides of a quadrilateral are equal, it is a parallelogram.

2. If the middle point of the base of an isosceles triangle be joined to the vertex, the joining line bisects the vertical angle, and is at right angles to the base.

EXAMPLES ON (I. 26).

1. Prove that the diagonals of a parallelogram bisect each other.

2. Prove that the bisectors of the base angles of an isosceles triangle are equal.

3. If a point be taken equally distant from two parallel lines, prove that all straight lines passing through this point, and intercepted between the parallels, are bisected at the point.

MISCELLANEOUS EXAMPLES.

1. Prove that the sum of the sides of a quadrilateral is greater than the sum of its diagonals.

[By applying (I. 20) to the four triangles of the quadrilateral, it will be found that double the sum of the sides is greater than double the sum of the diagonals.]

2. Prove that if any point be taken within a triangle, the sum of the lines joining the point to the vertices is less than the sum of the sides of the triangle.

[Consider I. 21.]

3. Prove, by showing that the angle *BDC* is greater than the angle *BCD* in fig. Prop. 13, that the difference of any two sides of a triangle is less than the third.

4. Prove that the angle included between any two straight lines is equal to the angle included between any other two lines drawn parallel to them.

5. If the sum of two angles of one triangle be equal to the sum of two angles of another, prove that the remaining third angles in each are equal.

[Consider Prop. I. 32.]

6. Prove that the angle of an equilateral triangle is one-third of two right angles, or two-thirds of one.

7. Hence, trisect a right angle.

8. Prove that if each base angle of an isosceles triangle be

double the vertical angle, this latter is two-fifths of a right angle.

9. Prove that the diagonals of a rhombus bisect its angles, and that they are at right angles to each other.

10. Of all parallelograms having the same diagonals, prove that the rhombus has the maximum area.

[Consider the preceding Example and Prop. 73.]

11. Inscribe a rhombus in a given triangle, having two of its sides coincident with an angle of the triangle.

[Bisect any one of the angles, and where the bisector meets a side, draw parallels to the other two sides: the parallelogram thus formed is a rhombus.]

12. Inscribe a square in a right-angled triangle, having two of its sides coincident with the right angle.

[This is solved in the same way as the last.]

13. Prove that the rectangle contained by a side of this square and a line equal to the sum of the sides of the triangle is equal to the rectangle contained by the sides.

14. Inscribe a square in any triangle; and prove that the rectangle contained by a side of the square and a line equal to the sum of the base and altitude of the triangle is equal to the rectangle contained by the base and altitude.

[Consider Ex. 12 and Prop. 4.]

15. Prove that the parallelogram *BDEF*, in Prop. 1, is the greatest that can be inscribed in the triangle *ABC*.

16. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the bisectors.

[Since (12) the sum of the squares of the sides is equal to twice the square of half the base plus twice the square of the bisector of that base, we have, if a, b, c, l, m, n, are the sides and respective bisectors,

$$b^2+c^2=\frac{a^2}{2}+2l^2,$$

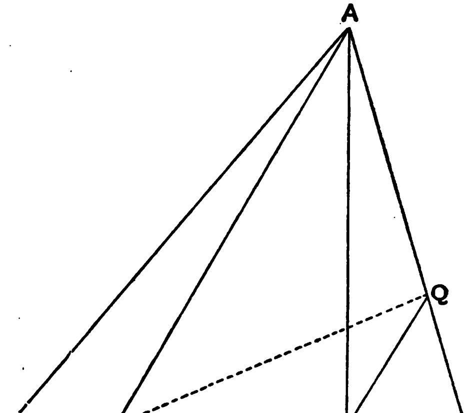
$$c^2+a^2=\frac{b^2}{2}+2m^2,$$

$$a^2+b^2=\frac{c^2}{2}+2n^2.$$

Therefore, by addition, $2(a^2+b^2+c^2)=\frac{a^2+b^2+c^2}{2}+2(l^2+m^2+n^2)$,

that is, $3(a^2+b^2+c^2)=4(l^2+m^2+n^2)$.]

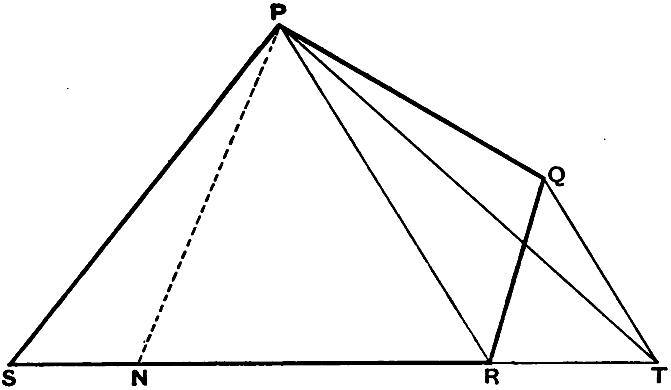
17. Through a given point in the base of a triangle draw a straight line cutting off the n^{th} part of the triangle.



[Let P be the point in the base, BC, of the triangle ABC. Divide (18) BC into n equal parts; let CN be one of these equal parts; join PA, NA, and draw NQ, parallel to PA, meeting AC in Q; join PQ: the triangle PCQ is the n^{th} part of the triangle ABC. For, clearly, the triangles NAQ, NPQ are equal (I. 37), and therefore the triangles PCQ, NAC are equal; but the triangle NAC is obviously the n^{th} part of the triangle ABC, since NC is the n^{th} part of the base BC, and therefore etc.

Note. When N bisects BC, the triangle PQC is half the triangle ABC. And when CN is the third part of BC, the triangle PQC is the third part of ABC; and so on.]

18. Through a vertex of a quadrilateral draw a straight line cutting off the n^{th} part of the quadrilateral.



[Let P be the vertex of the quadrilateral $PQRS$ through which it is required to draw a straight line cutting off its n^{th} part; draw QT, parallel to PR, meeting SR in T; join PT; take SN the n^{th} part of ST (18), and join NP: the triangle PNS is the required n^{th} part. For, the triangles PRT, PQR are equal, being on the same base and between the same parallels, therefore, adding to each the triangle PRS, the quadrilateral $PQRS$ is equal to the triangle PST; but the triangle PSN is the n^{th} part of the triangle PST, since SN is the n^{th} part of ST, therefore it is the n^{th} part of the quadrilateral.

Note. If SN is half of ST, the triangle PSN is half of the quadrilateral. And if SN is the third of ST, the triangle PSN is the third of the quadrilateral; and so on.]

19. Given the hypotenuse of a right-angled triangle, construct it so that the three sides shall be in continued proportion.

[Let (see fig. Prop. 6) AB be the given hypotenuse, and suppose that ABC is the required triangle, that is, that $AB \cdot BC = AC^2$. Now, if CD is perpendicular to AB, we know (32) that $AB.AD=AC^2$; therefore $AB.AD=AB.BC$; hence AD, BC must be equal. Therefore, since $AB.BD=BC^2$, $AB.BD$ must equal AD^2, in other words, AB is cut as in (II. 11). The solution is, therefore, as follows:—Divide AB in D, so that $AB.BD$ shall equal AD^2; upon AB describe a semicircle; let DC, perpendicular to AB, meet the circumference in C; join CA, AC: ABC must be the required triangle, as the pupil will now easily prove.]

20. Show that the locus of the centres of all circles of a given radius touching a fixed circle is a concentric circle.

21. Describe a circle of a given radius touching two given circles.

22. Describe a circle of a given radius touching a given circle and a given straight line.

23. Two chords of a given circle intersect at right angles; prove that the sum of the squares of the four segments is equal to the square of the diameter.

[Draw perpendiculars to the chords from the centre of the circle, when the problem is manifest from (II. 9).]

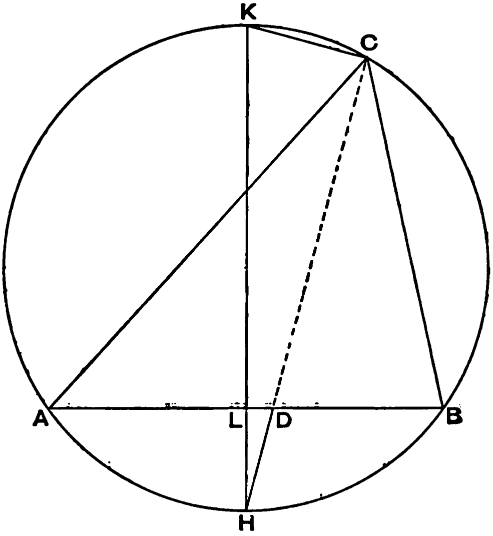
24. Given, of a triangle, the base, the bisector of the base, and the vertical angle: construct it.

[Since the base and vertical angle are given, the locus of the vertex is known, Prop. 45; and since the bisector of the base is given, the locus of the vertex is a circle, with the middle point of the base as centre, and a radius equal the given bisector. The point where these loci intersect is the vertex of the triangle.]

25. Given, of a triangle, the base, the bisector of the internal vertical angle, and the vertical angle: construct it.

[If ABC is the required triangle, and if CD, the internal bisector of the vertical angle, meet the circumference of the circumscribing circle again in H, and if HK, the diameter of the circle, meet the base, AB, in L, then $HL.HK$ is a fixed rectangle. But $CH.HD$ is equal to $HL.HK$, Prop. 17;

therefore $CH \cdot HD$ is given, and the difference of CH, HD is also given, viz., CD; therefore CH, HD can be determined, Prop. 33. But H is a fixed point; hence the vertex C is determined, and therefore the triangle ABC.]



26. Given, of a triangle, the base, the external bisector of the vertical angle, and the vertical angle: construct it.

[The solution here is the same as in the last, remembering that the points H, K interchange places.]

27. Through a given point, situated either on the internal or external bisector of an angle, draw a straight line, so that the segment intercepted between the legs of the angle shall be of a given length.

[By means of the foregoing examples, the problem is reduced to (I. 23).]

28. From any point on the circumference of a circle perpendiculars are drawn to two fixed diameters; prove that the line joining their feet is always of a constant length.

[A circle will clearly (15, 16) pass through the centre of the circle, the feet of the perpendiculars, and the variable point on the circumference. But the diameter of this latter circle is obviously equal to the radius of the given circle; therefore this circle is of a given magnitude; and since the angle between the diameters is fixed, the chord subtending this angle is therefore always constant (III. 26).]

29. A straight line of given length slides between two fixed straight lines; find the locus of the intersection of the perpendiculars erected to the fixed lines at the points where the sliding line meets them.

[The circumscribing circle of the quadrilateral formed by the fixed straight lines and the perpendiculars is clearly given, since the variable line is of given length and it subtends a constant angle at the intersection of the fixed lines. Hence the diameter of this circle is of given length. The locus is, therefore, a circle with the intersection of the fixed lines as centre, and a radius (the diameter of the constant circumscribing circle) of given length.]

30. Given, of a triangle, the base, in magnitude and position, and the vertical angle: construct it so that its vertex shall lie on a fixed straight line. When would there be only one solution?

31. Prove that the circumscribing circle of a triangle and the three circles, each passing through the intersection of the perpendiculars and the extremities of a side, are equal to one another.

32. Through a given point outside a circle a straight line is drawn cutting it in two points; draw through the point another straight line cutting the circle, so that, of the intercepted arcs—

(1) Their sum shall be given.

(2) Their difference shall be given.

33. Given, of a triangle, the base, vertical angle, and foot of the perpendicular drawn from the vertex to the base: construct it.

34. If a quadrilateral circumscribe a circle, prove that the sum of one pair of opposite sides is equal to the sum of the other pair.

35. Draw a straight line cutting two given circles, so that the intercepted chords shall each be of a given length.

[This is reduced to draw a common tangent to two given circles. See Props. 90, 84.]

36. If two circles cut (or touch), prove that their common chord (or the tangent at the point of contact) bisects the common tangent.

[This follows immediately from (III. 36).]

37. If two circles touch, their common tangent subtends a right angle at the point of contact.

38. If two circles touch, and through their point of contact any secant be drawn, prove that it divides the circles into similar segments.

39. If from the extremities of a diameter, AB, any two chords AH, BG be drawn in the same semicircle and meeting in C, prove that

$$AH \, . \, AC + BG \, . \, BC = AB^2.$$

[Clearly BH, AG are two of the perpendiculars of the triangle ABC. If CI be the third perpendicular, then (15, 16) the quadrilateral $BIHC$ is circumscribable by a circle;

therefore (17) $AH \, . \, AC = AI \, . \, AB.$

Similarly $BG \, . \, BC = BI \, . \, BA$;

$\therefore AH \, . \, AC + BG \, . \, BC = AB \, . \, AI + BI \, . \, BA = AB^2$ (II. 2).]

40. If through either point of intersection of two given circles any secant be drawn, prove that the triangle, having the portion of the secant intercepted between the circumferences as base, and the other point of intersection as vertex, has each angle constant.

41. Through either point of intersection of two given circles draw a secant, so that, of the intercepted chords—

(1) Their sum shall be equal to a given line.

(2) Their sum shall be a maximum.

[By means of the preceding Example the solution of (1) follows without any difficulty. To solve (2), it must be understood that of all equiangular triangles that, which has the greatest base, has also the greatest sides. This the pupil can easily satisfy himself upon. It, therefore, follows that the sum of the intercepted chords is a maximum when the sides of the triangle formed by this sum as base, and the other point of intersection as vertex, are diameters of the respective circles.]

42. Given the radii of the inscribed and circumscribed circles of a triangle, and an angle : construct it.

[Since an angle and the circumscribed circle of the triangle are given, a side must be known (III. 26). The problem is therefore reduced to Prop. 67.]

43. Given, of a triangle, the base, either of the base angles, and the distance between the centre of the inscribed circle and the centre of the escribed touching the base : construct it.

[Consider Prop. 23, Ex. 3.]

44. Prove that the diagonals of an equilateral and equiangular pentagon cut each other as in (II. 11).

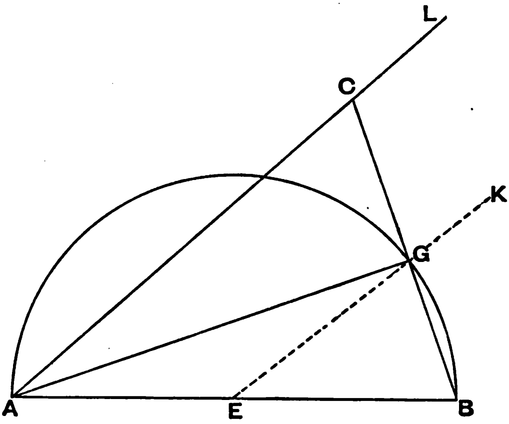
[For the solution of this consider Prop. 31.]

45. Divide a right angle into five equal parts.

46. Prove that the side of an equilateral and equiangular hexagon inscribed in a circle is equal to the radius.

47. *ABCDE* is an equilateral and equiangular pentagon inscribed in a circle: if the diagonals *AC*, *BD* intersect in *F*, prove that the points *F*, *O*, *E* are in the same straight line, *O* being the centre of the circle.

48. Given, of a triangle, the base, either base angle, and the ratio of the segments of the opposite side made by the perpendicular from the given base angle: construct it.



[Let *AB* be the given base, *BAL* the given base angle; divide *AB* in *E*, so that *AE* : *EB* in the given ratio of the required segments; through *E* draw *EK* parallel to *AL*, and cutting the semicircle described upon *AB* as diameter in *G*; join *BG*, and let *BG* produced meet *AL* in *C*: *ABC* is the required triangle. For, clearly, *AE* : *EB* :: *CG* : *GB* (VI. 2); and the angle *AGB* is right, being in a semicircle.

The solution of the above clearly solves the problem in Trigonometry—"To divide an angle into two parts so that the ratio of their tangents shall be given."]

49. Find the locus of a point, the ratio of the distances of which from two fixed lines shall be in a given ratio.

[Form any isosceles triangle having the point where the fixed lines meet as vertex, and portions of the fixed lines for sides; divide the base of this triangle into segments, the ratio of which shall be equal to the given ratio; the straight line joining this point of section to the intersection of the fixed lines is the required locus. The above solution evidently solves the problem—"To divide an angle into two parts so that the ratio of their sines shall be given."]

50. Find the locus of a point P, such that, if perpendiculars PQ, PR be drawn to two fixed lines ML, MN, the ratio $MQ : MR$ shall be in a given ratio.

[Take off portions NM, LM so that they shall have the given ratio; join NL; draw MP perpendicular to NL. It will be found that the line MP is the required locus.

The above solution evidently solves the problem—"To divide an angle into two parts so that their cosines shall be in a given ratio."]

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